NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1382

The Motion of (48) Doris and the Mass of Jupiter

J. William Zielenbach

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Preface

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Abstract

A definitive orbit is obtained for (48) Doris based upon the provisional reciprocal mass of Jupiter. Numerically integrated variational equations for the coordinates of Doris with respect to its initial rectangular coordinates and velocities and the mass of Jupiter are used to differentially correct the orbit of Doris and the mass of Jupiter. The reciprocal mass obtained, using 617 observations over a 110-yr time span, is 1047.340 ± 0.016.

The Motion of (48) Doris and the Mass of Jupiter

I. Introduction

Jupiter is the most massive planet in the solar system and has an important gravitational influence on the motion of all other bodies in the solar system. Since the gravitational effect is dependent upon the mass of Jupiter, this mass must be determined for accurate representation of planetary and interplanetary motions.

The mass of the planet itself is not easily determined from the motion of its satellites because the size and shape of the disk make it difficult to measure their positions with respect to the center of the planet. The interaction of the inner satellites is quite complicated, and relatively few observations have been made of the outer, less perturbed satellites. Consequently, investigators have tended to analyze the gravitational effect of the whole Jupiter system at interplanetary distances.

Newcomb (Ref. 1) pointed out that perhaps the best determination of the mass by classical astronomical methods would be that afforded by the motion of minor planets (asteroids) because their positions can be more precisely observed than can those of comets or major planets. Moreover, the general location of the belt of asteroids between Mars and Jupiter makes the asteroids highly susceptible to Jovian perturbations. The paragraphs that follow indicate the reasons why certain asteroids are more suitable than others for determining the mass of Jupiter by classical astronomical methods.

From the theory of general perturbations (Ref. 2, p. 467), it is known that the disturbing function $\mathcal{R}$ for any object perturbed by a body of mass m' is given by

$$\mathcal{R} = k^2 \frac{m'}{m_\odot + m'} \sum_{j,k,m,j',k',m'} F(a,a',e,e',i,i')$$
$$\cos(j\Omega + k\omega + mM + j'\Omega' + k'\omega' + m'M')$$

where a, e, i, Ω, ω, and M are the usual Keplerian elements for the perturbed body and the primed quantities are the elements of the perturbing planet. The analytical expressions for the time variations of the elements of the perturbed body are obtained by integration. Since the mean anomaly M can be written as

$$M = nt + \sigma$$

where n is the mean motion and t is the time elapsed since M equaled σ, the trigonometric term can be written

$$\cos\left[(jn + j'n')t + \theta(j,j',k,k',m,m',\Omega,\Omega',\omega,\omega',\sigma,\sigma')\right]$$

which, when integrated, will have a term involving $(jn + j'n')$ in the denominator. If there is a near commensurability of the mean motions n and n' for indices j and j', the resulting coefficient of the theory is large and the period of the trigonometric term is long.

In 1873, G. W. Hill (Ref. 3) drew attention to the fact that the Hecuba group of minor planets has nearly 2:1 commensurabilities of mean motions with Jupiter. This gives rise to periodic perturbations of large amplitudes, whose periods are short enough to be observed within a reasonable length of time. He pointed out that, since proximity to Jupiter greatly affects the magnitude of the perturbations, asteroids with large semimajor axes, as well as highly elliptic orbits whose aphelia are near Jupiter, would be most desirable (as long as the mutual inclination of the two orbits is small). These criteria are fulfilled to a greater or lesser degree by the 13 minor planets he recommended for future observation and analysis.

Minor planet (48) Doris is one of this group. Its long period term is about 72 yr, and there has been ample opportunity to observe it since its discovery in 1857. The perturbation in longitude has an amplitude slightly under 1.5°, making it the least affected of the 13 asteroids.

This report contains a study of the motion of (48) Doris and a numerical analysis of the effect of Jupiter upon this motion.

Variational equations with respect to the rectangular starting coordinates and the mass of Jupiter were obtained for the coordinates of the minor planet by numerical integration. A definitive reference orbit was obtained by differentially correcting numerically integrated orbits, using 617 observations. The resultant reciprocal mass for the Jupiter system, as determined by these observations, is 1047.340 ±0.016.

The sections that follow include descriptions of the reduction techniques for putting all of the observations on a common system, the numerical integration of the equations of motion and their partial derivatives, the method and statistics of the solution of the conditional equations, and the formation of the differential correction coefficients. The input observations and final results are critically analyzed, and the various computational aspects of the problem are discussed with the benefit of hindsight and with an eye to future research.

II. Numerical Integration

Numerical solution of differential equations has become common with the advent of electronic computers. This is especially true for cases of perturbed motion for which no complete analytical formulation is available. A typical example is the calculation of planetary orbits by the method of special perturbations. This section begins with a description of the basic differential equation of motion to be integrated, along with its partial derivatives with respect to various parameters. It concludes with a brief description of the techniques used in the integration and a presentation of general starting conditions.

The equation of motion and its derivatives have been expressed in a center-of-mass (c.m.) frame because the amount of computation required to evaluate $\ddot{\mathbf{r}}$ for one object perturbed by n planets and the sun is roughly $(n+1)/(2n+1)$ of that required in heliocentric coordinates. Transforming any quantity from barycentric to heliocentric merely involves subtracting the appropriate barycentric value for the sun. Depending upon the number of equations being integrated and the bodies to which they refer, it is sometimes more efficient, however, to integrate the heliocentric variational equations with respect to the starting coordinates because the orbits being corrected are traditionally heliocentric.

A. Equations To Be Integrated

By considering the magnitude of the effects of general relativity and oblateness of perturbing bodies upon the orbit of (48) Doris, it can be seen that the motion of this minor planet is adequately described by a nonrelativistic point mass equation of motion. The resulting expression for a body with mass m_i, acted upon by n other bodies of mass m_j, is given in the c.m. system by

$$\frac{d^2}{dt^2}\mathbf{r}_i = \frac{d}{dt}\dot{\mathbf{r}}_i = \ddot{\mathbf{r}}_i = -k^2 \sum_{j \neq i} m_j \frac{(\mathbf{r}_i - \mathbf{r}_j)}{\rho_{ij}^3} \tag{1}$$

where $\mathbf{r}_i$ and $\mathbf{r}_j$ are barycentric coordinate vectors of the bodies with mass m_i and m_j, and $\rho_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. The expression is just Newton's law of gravitation, where k is the Gaussian constant 0.01720209895 $(\mathrm{AU}^3/\mathrm{day}^2$ solar masses$)^{1/2}$.

The major effect of considering relativity was found to be a 0″.2285/century advance of the perihelion of (48) Doris. Because this value is negligible in comparison with the errors of observation, Eq. (1) was deemed sufficient for generating the orbit of (48) Doris. Relativistic effects are important for the earth, but, as will be seen below, are already included in the ephemerides of that body. The perturbative effect of oblate bodies was also considered. The objects most liable to affect (48) Doris in this regard are the sun and Jupiter. However, their influence can be neglected.

The conclusion that neither the sun nor Jupiter causes a significant oblateness perturbation is based upon the calculations that follow.

Define the oblateness Δ of an object in terms of its equatorial and polar radii r_e and r_p by $\Delta = (r_e - r_p)/r_p$. With Dicke's value (Ref. 4) of $\Delta = 5 \times 10^{-5}$ as an upper limit for the sun, the predicted centennial perihelion advance of (48) Doris is 0″.0027, whereas that of the earth is 0″.1403. The total effect upon the position of (48) Doris is far below the errors of observation. An upper limit for the effect of Jupiter is obtained by letting (48) Doris orbit that body at the distance of its closest approach to the planet—roughly 2 astronomical units (AU). The resulting centennial advance caused by a Jovian oblateness of 0.062 is less than 0″.00015.

In view of the requirements of differential correction processes, it is desirable to consider the partial derivatives of the equation of motion with respect to parameters whose values might be improved. Since k is invariable by convention, Eq. (1) is explicitly a function only of masses and coordinates. Partial derivatives for each of these quantities are developed in the paragraphs that follow.

If an improved value for the mass of a planet m'_j can be so written in terms of an existing mass m_j by means of a correction factor $(1 + \theta_j)$ that

$$m'_j = (1 + \theta_j) m_j \tag{2}$$

then the partial derivative of Eq. (1) with respect to θ_j is

$$\frac{d^2}{dt^2}\frac{\partial \mathbf{r}_i}{\partial \theta_j} = \frac{d}{dt}\frac{\partial \dot{\mathbf{r}}_i}{\partial \theta_j} = \frac{\partial \ddot{\mathbf{r}}_i}{\partial \theta_j} + \frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_i}\frac{\partial \mathbf{r}_i}{\partial \theta_j} + \sum_{k \neq j, i} \frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_k}\frac{\partial \mathbf{r}_k}{\partial \theta_j} \tag{3}$$

The quantities $\partial \ddot{\mathbf{r}}_i / \partial \mathbf{r}_i$, $\partial \ddot{\mathbf{r}}_i / \partial \mathbf{r}_k$, and $\partial \ddot{\mathbf{r}} / \partial \theta_j$ are given in Eq. (4). (It should be noted that the derivative of a vector with respect to a vector is introduced as a notational convenience only.)

$$\Delta x = \mathbf{x}_i - \mathbf{x}_j, \qquad \Delta y = \mathbf{y}_i - \mathbf{y}_j, \qquad \Delta z = \mathbf{z}_i - \mathbf{z}_j \tag{4a}$$

$$\rho = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} \tag{4b}$$

$$\frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_i} = -k^2 \begin{vmatrix} \sum_j m_j \left(\frac{3\Delta x^2}{\rho^5} - \frac{1}{\rho^3}\right) & 3\sum_j m_j \frac{\Delta x \Delta y}{\rho^5} & 3\sum_j m_j \frac{\Delta x \Delta z}{\rho^5} \\ 3\sum_j m_j \frac{\Delta y \Delta x}{\rho^5} & \sum_j m_j \left(\frac{3\Delta y^2}{\rho^5} - \frac{1}{\rho^3}\right) & 3\sum_j m_j \frac{\Delta y \Delta z}{\rho^5} \\ 3\sum_j m_j \frac{\Delta z \Delta x}{\rho^5} & 3\sum_j m_j \frac{\Delta z \Delta y}{\rho^5} & \sum_j m_j \left(\frac{3\Delta z^2}{\rho^5} - \frac{1}{\rho^3}\right) \end{vmatrix} \tag{4c}$$

$$\underset{j \neq i}{\frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_j}} = +k^2 m_j \begin{vmatrix} \left(\frac{3\Delta x^2}{\rho^5} - \frac{1}{\rho^3}\right) & \frac{3\Delta x \Delta y}{\rho^5} & \frac{3\Delta x \Delta z}{\rho^5} \\ \frac{3\Delta y \Delta x}{\rho^5} & \left(\frac{3\Delta y^2}{\rho^5} - \frac{1}{\rho^3}\right) & \frac{3\Delta y \Delta z}{\rho^5} \\ \frac{3\Delta z \Delta x}{\rho^5} & \frac{3\Delta z \Delta y}{\rho^5} & \left(\frac{3\Delta z^2}{\rho^5} - \frac{1}{\rho^3}\right) \end{vmatrix} \tag{4d}$$

$$\frac{\partial \ddot{\mathbf{r}}_i}{\partial \theta_j} = -k^2 m_j \frac{\mathbf{r}_i - \mathbf{r}_j}{\rho^3} \tag{4e}$$

Partial derivatives with respect to the coordinates can also be written explicitly, but other considerations should be introduced to render them applicable for the differential correction of orbits.

Because the position and velocity of an object are obtained by integrating differential equations, their values at any time depend ultimately upon the integration constants which are related to the boundary conditions satisfied by the differential equations. It follows that—if the functional expression of, and independent variables in, the differential equations remain unchanged—the only means of generating a different orbit using the equations is to modify the starting constants. For orbit correction, then, it is desirable to have expressions showing the dependence of the position and velocity (at any time) upon the initial position and velocity. In general, because some of the independent variables (namely, the $\mathbf{r}_j$ variables) depend upon their own starting conditions, it is conceivable to relate $\mathbf{r}_i$ to the starting coordinates of any object m, including itself. If $\mathbf{u}_{m_0} = (\mathbf{r}_{m_0}, \dot{\mathbf{r}}_{m_0})$, then, by differentiating Eq. (1) with respect to $\mathbf{r}_m$ and applying the chain rule,

$$\begin{aligned}\frac{d^2}{dt^2}\frac{\partial \mathbf{r}_i}{\partial \mathbf{u}_{m_0}} &= \frac{d^2}{dt^2}\frac{\partial \mathbf{r}_i}{\partial \mathbf{r}_m}\frac{\partial \mathbf{r}_m}{\partial \mathbf{u}_{m_0}} \\ &= \frac{d}{dt}\frac{\partial \dot{\mathbf{r}}_i}{\partial \mathbf{u}_{m_0}} = \frac{d}{dt}\frac{\partial \dot{\mathbf{r}}_i}{\partial \mathbf{r}_m}\frac{\partial \mathbf{r}_m}{\partial \mathbf{u}_{m_0}} \\ &= \frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_i}\frac{\partial \mathbf{r}_i}{\partial \mathbf{r}_m}\frac{\partial \mathbf{r}_m}{\partial \mathbf{u}_{m_0}} + \sum_{k\neq m,\, i}\frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_k}\frac{\partial \mathbf{r}_k}{\partial \mathbf{r}_m}\frac{\partial \mathbf{r}_m}{\partial \mathbf{u}_{m_0}} \\ &= \frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_i}\frac{\partial \mathbf{r}_i}{\partial \mathbf{u}_{m_0}} + \sum_{k\neq m,\, i}\frac{\partial \ddot{\mathbf{r}}_i}{\partial \mathbf{r}_k}\frac{\partial \mathbf{r}_k}{\partial \mathbf{u}_{m_0}} \end{aligned} \qquad (5)$$

Eq. (4) contains mathematical definitions of the terms involved.

As no attempt was made to improve the orbit of any planet by means of perturbation effects upon (48) Doris, Eq. (5) was not used for $m \neq i$. When $m = i$, and i is considered massless, the cross terms become meaningless and may be neglected.

B. Method of Integration

The numerical integration of these differential equations can be accomplished by a variety of techniques. In this report, a method derived from the Lagrangian interpolation polynomial was used to start the integrations, whereas a modified backward difference approach was used for extrapolation. Both techniques can be used for single or double integration of the function $f(t)$, whose tabular values are defined by

$$f(t_i) = \frac{d}{dt}g(t_i) = \frac{d^2}{dt^2}h(t_i) \qquad (6)$$

The Lagrangian interpolation formula (Ref. 5) expresses the value of a function $f(t)$ at any point $t = \tau$ to within some error $R_n(\tau)$ by

$$f(\tau) = \sum_{i=-n/2}^{+n/2} \ell_i(\tau) f(t_i) + R_n(\tau) \qquad (7)$$

where

$$\begin{aligned}\ell_i(\tau) &= \frac{\Pi_n(\tau)}{(t - t_i)\Pi_n(t_i)} \\ &= \frac{(\tau - t_0)\cdots(\tau - t_{i-1})(\tau - t_{i+1})\cdots(\tau - t_n)}{(t_i - t_0)\cdots(t_i - t_{i-1})(t_i - t_{i+1})\cdots(t_i - t_n)}\end{aligned} \qquad (8)$$

Let $F(\tau)$ represent the literal polynomial given by the summation term in Eq. (7), when τ is indefinite. The first and second integrals of $F(\tau)$, denoted $F^1(\tau)$ and $F^2(\tau)$, are defined by

$$F^1(\tau) = \sum_i L_i^1(\tau) f(t_i) + C^1 \qquad (9)$$

$$F^2(\tau) = \sum_i L_i^2(\tau) f(t_i) + C^1\tau + C^2 \qquad (10)$$

where L^1 and L^2 are the corresponding integrals of ℓ in Eq. (7), and C^1 and C^2 are integration constants. The value of the desired integral F^k at any point τ in terms of its value at any other point τ' is simply $F^k(\tau) - F^k(\tau')$. If it is assumed that $g(t_i) \simeq F^1(t_i)$ and $h(t_i) \simeq F^2(t_i)$, Eqs. (9) and (10) provide a means for integrating Eq. (6).

In practice, the initial conditions $g(t_0)$ and $h(t_0)$ define $F^1(t_0)$ and $F^2(t_0)$, and thereby define the integration constants. The source of the initial conditions for the various equations is discussed below. Because the nonrelativistic equation of motion was chosen, the expression for the acceleration does not involve the velocity. Also, none of the other functional expressions for the second derivatives involves the first derivatives. This means that it is possible to use Eq. (10) iteratively to obtain converged values for each $h(t_i)$, and then apply Eq. (9) once to determine each $g(t_i)$.

Means will be mentioned below for computing approximate values $h(t_i)$ from which $f(t_i)$ is derived. The complete set of $n + 1$ points $f(t_i)$ can be used in Eq. (10) to estimate some new $h(t_i)$, which redefines $f(t_i)$. The process is applied for n terms $h(t_k)$ in the order $k = +1, -1, +2, \cdots +n/2, -n/2$, and iterated to an arbitrary criterion of convergence. Because $h(t_0)$ is an epoch condition, it remains unaltered; it could never be changed by Eq. (10) since

$$\sum_i L_i^2(t_0)f(t_i) = \sum_i L_i^1(t_0)f(t_i) = 0 \tag{11}$$

The converged points $h(t_i)$ imply converged values of $f(t_i)$, from which each $g(t_i)$ can be derived by Eq. (9).

The starting conditions for the equation of motion are the epoch position and velocity in the appropriate frame of reference. The initial partial derivatives of barycentric coordinates with respect to a mass factor θ_j are obtained by differentiating the expression for the c.m. with respect to θ_j. In the c.m. system, if $\mathbf{U}_c$ represents the coordinates or any of their time derivatives, then

$$\frac{\partial \mathbf{U}_c}{\partial \theta_j} = \frac{\partial}{\partial \theta_j} \frac{\sum (1+\theta_i)\mathbf{U}_i m_i}{\sum (1+\theta_i) m_i} = \frac{m_j \mathbf{U}_j}{\sum (1+\theta_i) m_i} \tag{12}$$

The change in initial barycentric $\mathbf{U}_i$ of any object due to θ_j is just the negative of $\partial \mathbf{U}_c / \partial \theta_j$. In transforming to heliocentric, these quantities all become zero. The initial values for the variational equations with respect to coordinates and velocities are the same in any reference frame. By inspection of Eq. (5), for $m = i$,

$$\frac{\partial \mathbf{r}_i}{\partial \mathbf{r}_{i_0}} = \mathbf{I}, \frac{\partial \mathbf{r}_i}{\partial \dot{\mathbf{r}}_{i_0}} = \phi, \frac{\partial \dot{\mathbf{r}}_i}{\partial \mathbf{r}_{i_0}} = \phi, \frac{\partial \dot{\mathbf{r}}_i}{\partial \dot{\mathbf{r}}_{i_0}} = \mathbf{I} \Bigg|_{t=t_0} \tag{13}$$

where $\mathbf{I}$ and ϕ are the identity and null matrices, respectively. For $m \neq i$, all of the above expressions are ϕ.

As approximate values for the starting table of the equations of motion, two-body orbits can be used, computed from the f and g series or from osculating Keplerian elements. For the variational equations with respect to starting coordinates, the boundary conditions can be propagated throughout the table. An alternate approach is Goodyear's expressions (Ref. 6) for the two-body partial derivatives, in which the necessary position and velocity are obtained from the already converged perturbed orbit. The derivatives for the mass can be approximated sufficiently by the first term in Eq. (3).

The extrapolation procedures use backward difference techniques as follows: In the conventional difference operator notation, and by the use of the previous definition of f, g, and h, with integration stepsize t',

$$\nabla g(t_k) = \int f(t)dt = t' \left[\frac{-\nabla}{ln(1-\nabla)}\right] f(t_k) \tag{14}$$

and

$$\nabla^2 h(t_k) = \int\int f(t)dt = t'^2 \left[\frac{-\nabla}{ln(1-\nabla)}\right]^2 f(t_k) \tag{15}$$

These are *difference* rather than *sum* forms of the classical corrector formulas for single and double integration. The predictor formulas are obtained by applying the shift operator $E = (1-\nabla)^{-1}$ to the above:

$$E\nabla g(t_k) = \nabla g(t_{k-1}) = (1-\nabla)^{-1} t' \left[\frac{-\nabla}{ln(1-\nabla)}\right] f(t_k) \tag{16}$$

and

$$E\nabla^2 h(t_k) = \nabla^2 h(t_{k-1}) = (1-\nabla)^{-1} t'^2 \left[\frac{-\nabla}{ln(1-\nabla)}\right]^2 f(t_k) \tag{17}$$

The backward differences can be expressed in terms of the tabular values of f, using the binomial coefficients, so that the final equations involve coefficients only of f.

The formulas actually used for Eqs. (14) through (17) are of the nth order for the predictor and $(n+1)^{\text{th}}$ for the corrector:

$$g(t_k) = g(t_{k-1}) + \sum_{i=-1}^{-(n+1)} P_i f(t_{k+i}) \tag{18}$$

$$h(t_k) = 2h(t_{k-1}) - h(t_{k-2}) + \sum_{i=-1}^{-(n+1)} Q_i f(t_{k+i}) \tag{19}$$

$$g(t_k) = g(t_{k-1}) + \sum_{i=0}^{-(n+1)} R_i f(t_{k+i}) \tag{20}$$

$$h(t_k) = 2h(t_{k-1}) - h(t_{k-2}) + \sum_{i=0}^{-(n+1)} S_i f(t_{k+i}) \qquad (21)$$

The coefficients P_i, Q_i, R_i, and S_i are those just described.

III. Numerical Integration (Details of Application)

The theory presented in Section II was implemented in an n-body program for numerical integration. Variational equations for any object, and derivatives of the coordinates of any object with respect to the mass of any other body, could be integrated simultaneously with the equations of motion. The option existed either to generate the ephemerides of the perturbing bodies or to assume them as input.

The integration of coordinates was checked by comparison with Refs. 7 and 8. The variational equations for the rectangular coordinates agreed satisfactorily with finite-difference partial derivatives (see Section VIII) and with the variational equations derived by other investigators. The form of the equations for these quantities was complete, involving no neglected terms other than the remainders always present in numerical integration.

The derivatives with respect to the mass of Jupiter were computed for (48) Doris and the earth, using the first two terms of Eq. (3). The cross terms have been assumed by other investigators to be negligible in view of the precision requirements of differential correction. It was hoped, at first, that these terms could be integrated and their behavior examined, but core storage limitations and the increased computer time were prohibitive. It may be possible that the accuracy resulting from inclusion of such second-order terms will never be necessary for analysis of visual observations.

The derivatives with respect to the mass, unlike the variational equations for the rectangular coordinates, are dependent upon the coordinate system. If the derivatives of the barycentric coordinates of the sun were being integrated, derivatives of the barycentric coordinates of (48) Doris and the earth could then be reduced to heliocentric, as mentioned in Section II; because this would require more computation than partial derivatives of the heliocentric coordinates, however, the latter approach was chosen. The technique was checked with that used by Lieske (Ref. 9).

The actual process of numerical integration could be of any arbitrary order. In view of core limitations, a method using eighth differences of the second derivatives was selected. This scheme was found to be sufficiently accurate over 110 yr to allow a 4-day interval to be used in the integration. The integration could be run with a predictor-only, or with a predictor–corrector arrangement that iterated to absolute convergence. With the shorter stepsize, it was hoped that the predictor-only arrangement would be sufficiently accurate for the purposes at hand since, on the average, it consumed less time than the predictor–corrector arrangement at 8 days.

Figure 1 compares a predictor-only run with a predictor–corrector run (4-day stepsize). Since (48) Doris orbits the sun at a distance of about 3 AU, the maximum difference represents 2×10^{-6} rad or about $0''.5$/century in heliocentric longitude, amounting to $0''.75$/century at the earth. This discrepancy, although a source of systematic error, was considered as admissible as the difference due to neglecting relativity. The contribution to Fig. 1 made by roundoff is shown in Fig. 2. Because of the secular runoff, it was difficult to compare the results with the $0.1184n^{3/2}$ last-place accumulated error predicted by Brouwer (Ref. 10) for n integration steps.

Presumably, the effect of both of the errors mentioned above would be ameliorated by choosing an epoch around 1910 and integrating forward and backward from that point. This approach was rejected in favor of the continuous backward integration because it was very difficult to reverse one or the other of the integrated output tapes on the computer.

Neither the variational equations nor the derivatives with respect to mass were examined for this type of accuracy because it was felt that the differences would not noticeably affect the differential correction process. This was, in fact, the case. The differential improvement worked well even with theoretically approximate derivatives.

The n-body program was designed to enable computation of perturbation ephemerides that were dynamically consistent, as well as to permit simultaneous integration of numerous *massless* bodies with subsequent reduction of perturbation ephemeris input time. To integrate a dynamically consistent perturbation ephemeris required the choice of starting values for the major planets. Schubart and Stumpff (Ref. 11) have published a set of values for the planets Venus through Pluto, but it was

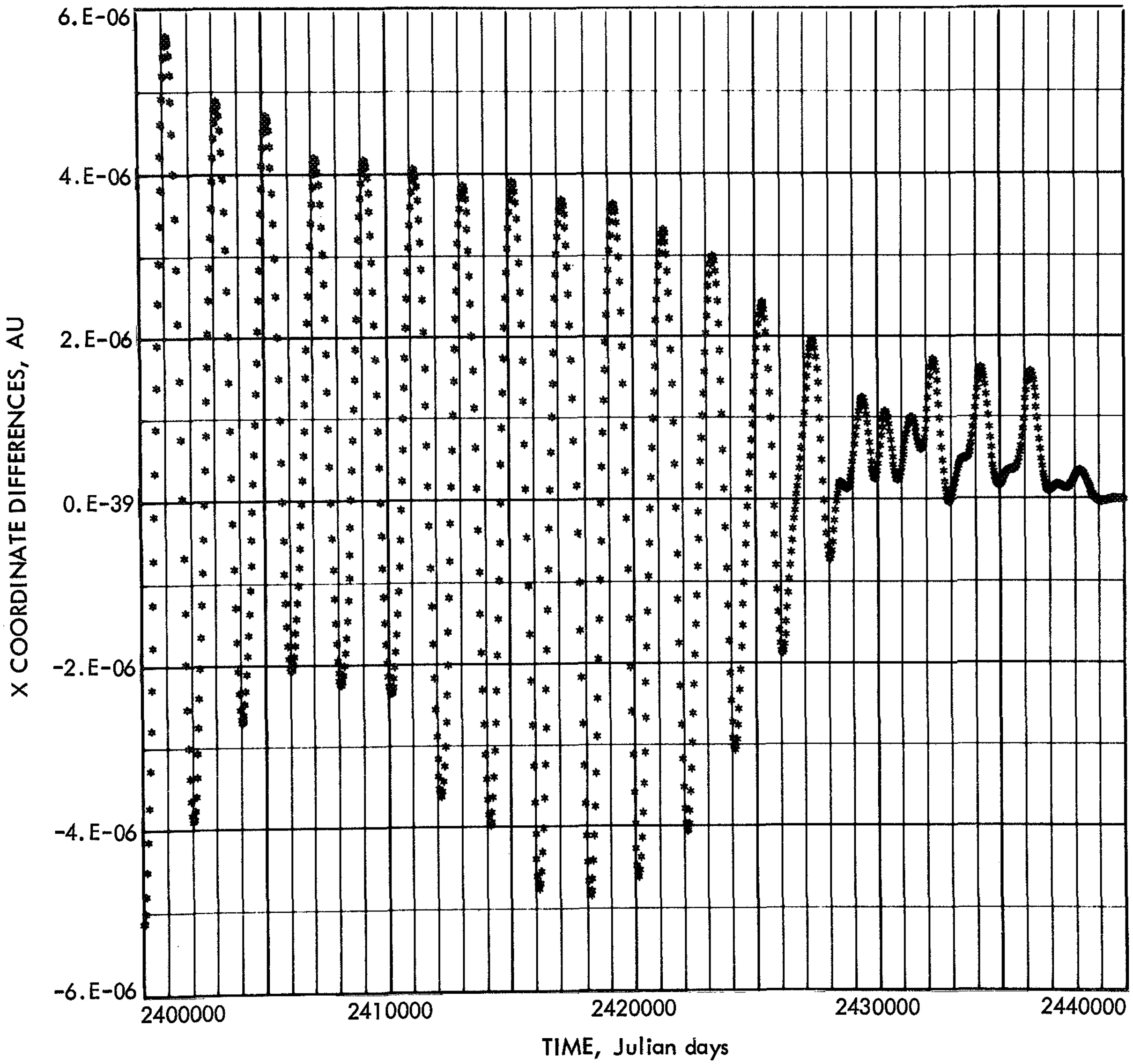

Fig. 1. Comparison of a predictor-only run with a predictor–corrector run, both of eighth order and 4-day stepsize

decided to incorporate Mercury in the work on (48) Doris. Lieske's Development Ephemeris 28 (see Ref. 8) is a Newtonian n-body integration fit to standard astronomical ephemerides of all nine planets; since it was available on tape, and had been used as a check for the integration described earlier, it was used as the actual input ephemeris.

In addition to the Newtonian ephemerides, Ref. 8 also contains differences between relativistic and Newtonian coordinates for all of the planets. It further includes the 7.″700/century effect of the acceleration of the moon on the earth–moon barycenter.

The tabular interval in Ref. 8 is 4 days. Because a predictor-only approach was used, this became the integration stepsize for (48) Doris. All integrated quantities were written at each step so that the input for the differential correction contained coordinates and partial derivatives at 4-day intervals. These were interpolated, as described in Section V, using an eighth-order Lagrangian formula (see Eq. 7).

The International Astronomical Union (IAU) system of planetary masses was used in Ref. 8 and in the studies of (48) Doris. Physical constants used are listed in Table 1, and values for the reciprocal masses appear in Table 2.

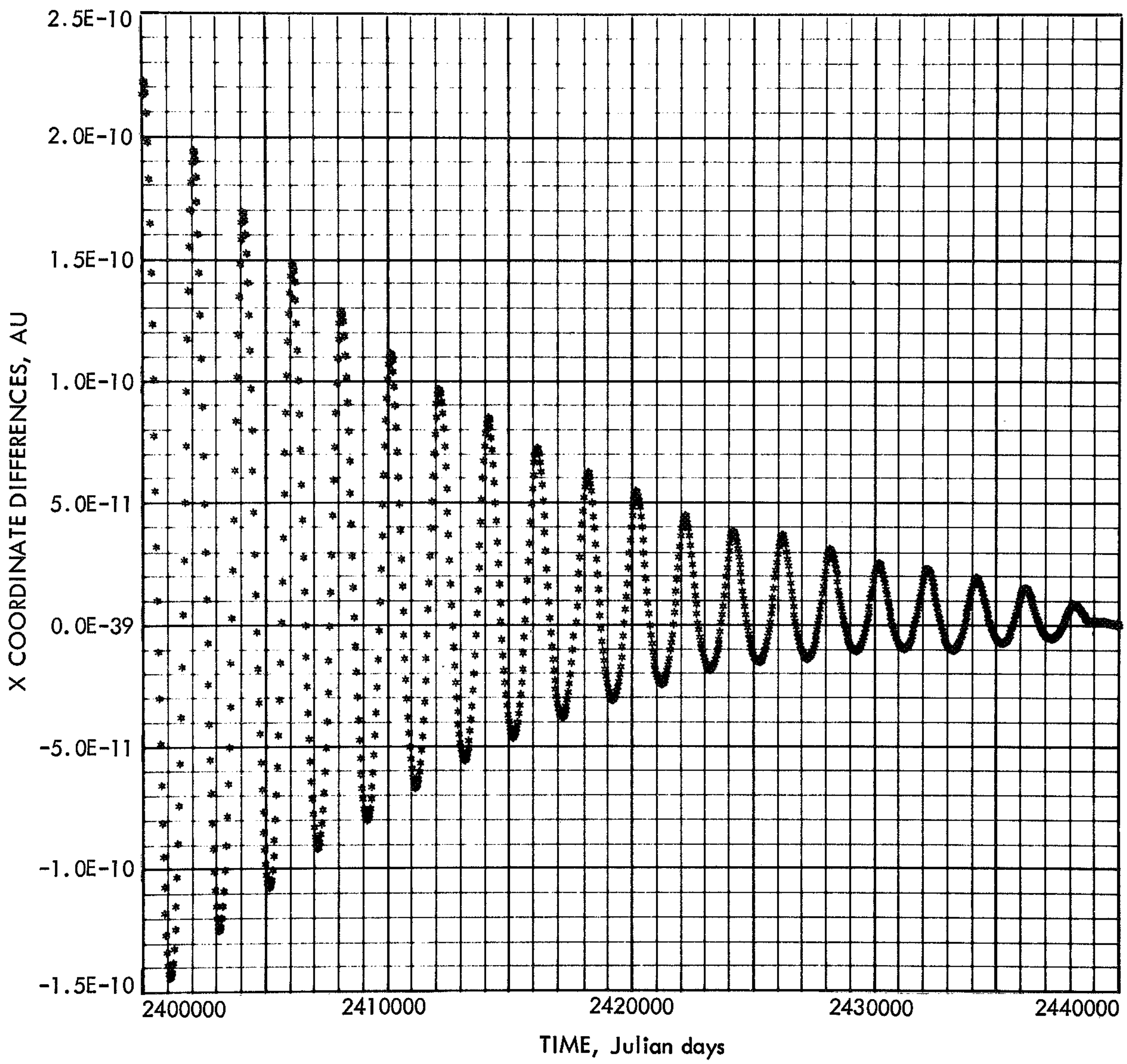

Fig. 2. Effect of stepsize—predictor–corrector run at 8 days, minus one at 4 days

Table 1. Physical constants

Constant	Symbol	Value
Obliquity at 1950.0	$\epsilon_{1950.0}$	$23°26'44''.836$
Aberration constant	k	$20''.4958$
Light time for 1 AU	$1/C_A$	$499^s.012 = 0.00577560185^d$
Equatorial radius of earth, km	a	6378.160
Flattening factor	f	298.25
Astronomical unit, km	AU	149,600,000
Annual rate of lunisolar precession on fixed ecliptic of date	ψ'	$50''.3708 + 0''.0050\ T$
Annual rate of planetary precession of date	λ'	$0''.1247 - 0''.0188\ T$
Eccentricity of earth	e	$0.01675104 - 0.00004180\ T - 0.000000126\ T^2$
Longitude of perihelion of earth	ϖ	$281°13'15''.00 + 6189''.03\ T + 1.63\ T^2 + 0''.012\ T^3$

Table 2. Reciprocal solar masses

Body	Reciprocal solar mass
Mercury	6,000,000
Venus	408,000
Earth–moon	329,390
Mars	3,093,500
Jupiter	1047.355
Saturn	3501.6
Uranus	22,869
Neptune	19,314
Pluto	3,600,000

IV. Least-Squares Differential Correction

A. Conditional Equations

The basic concept behind differential correction techniques is that the difference between an observed and a computed value can be represented by the derivative terms of a Taylor series in the parameters to be corrected, evaluated with approximate values of the parameters. For example: If a quantity F^* is represented by some function f of n parameters g_k for which approximate values g'_k are known, and an additional m parameters h_s whose values are known exactly, F^* may be expressed as

$$F^* = f(g'_1, \cdots g'_n, h_1, \cdots h_m) + \sum_k \left.\frac{\partial f(g,h)}{\partial g_k}\right|_{g'} \Delta g_k + \frac{1}{2}\sum_i \sum_k \left.\frac{\partial^2 f(g,h)}{\partial g_k \partial g_i}\right|_{g'} \Delta g_k \Delta g_i + \cdots \tag{22}$$

where $\Delta g_k = g_k - g'_k$. Because there is often reason to believe that the corrections Δg are small enough to warrant neglecting the higher-order terms, the series is usually truncated after the first order; the resulting linear expressions are then used to solve for n values Δg_k. Actually, because of the truncation, the solution yields some $\underline{\Delta g}_k$. The first term on the right side of Eq. (22) gives some value F', so that each member of the set of linear equations to be solved is of the form

$$F^* - F' = \sum_k \left.\frac{\partial f(g,h)}{\partial g_k}\right|_{g'} \underline{\Delta g}_k \tag{23}$$

For the hypothetical case in which there are n such equations, and the correct set of parameters g_k is known from independent means, it is often possible to iterate the procedure until it converges to these values (as long as the original estimate for each parameter g'_k is sufficiently close to the correct value). The question of how close is sufficiently close depends upon the behavior of the partial derivatives $\partial f/\partial g_k|_{g'}$ as g'_k approaches g_k.

The effect of approximations in the formulation of the derivatives $\partial f/\partial g_k$ also depends upon the above mentioned behavior, as well as the degree to which the actual $\partial f/\partial g_k|_{g'}$ is represented by the approximation. The functions f presented in Section V fortunately allow some well-known approximations, which are described there. In Section VIII, the results obtained with formally correct derivatives are compared to those obtained with approximations.

B. Formation and Solution of Normal Equations

In reality, all physical quantities g_k are determined empirically; therefore, it is difficult (if not meaningless) to speak of correct, true, or absolute values for such parameters. Instead, one speaks of the most probable values for the set of parameters in view of the data being used. The data generally have some errors caused by a combination of factors, but the distribution of the errors is usually assumed to be the most probable one to be expected from the "correct" values of g_k. What appears to be circular reasoning simply states that, if the error distribution on the data is the most probable one, then theoretically the most probable values of the parameters determined from those data will be the "true" ones.

The procedure for determining the most probable value for a set of parameters is called *maximum likelihood estimation,* and is unbiased if the error distribution on the data is the most probable one. Gauss has shown that, if the distribution of errors on the data is normal, namely, of the form

$$\frac{he^{-h^2x^2}}{\pi^{1/2}} \tag{24}$$

where h is a measure of precision of the observations, then for overdetermined systems, the most probable set of values for the parameters is that which minimizes the sum of squares of residuals between the observed and computed values. Gauss further extended the concept to include the possibility of weighting individual observations, in which case the most probable values of the parameters are those that minimize the sum of squares of the weighted residuals.

A normal error distribution was assumed for the data used in this report. Thus, for each observation time t, there was some value of the function F_t^* based upon the most probable set of parameters g_k which was related to the observed value $\tilde{f}_t$ and the intrinsic error of the observation e_t by

$$F_t^* = \tilde{f}_t - e_t \tag{25}$$

This was represented by the conditional equation

$$w_t^{1/2}(\tilde{f}_t - e_t - F'_t) = w_t^{1/2} \sum_i^n \left. \frac{\partial f(g,h)}{\partial g_i} \right|_{g'_i} \Delta g_i \tag{26}$$

where the weight assigned to the observation is denoted by $w_t^{1/2}$. This may be written in matrix form as

$$w_t^{1/2}[q_t] = w_t^{1/2}[a_{t,1}, \cdots a_{t,n}] \begin{bmatrix} \Delta g_1 \\ \cdot \\ \cdot \\ \cdot \\ \Delta g_n \end{bmatrix} \tag{27}$$

The set of m such expressions may be represented by the matrix equation

$$\mathbf{W}^{1/2}_{(m\times m)} \mathbf{Q}_{(m\times 1)} = \mathbf{W}^{1/2}_{(m\times m)} \mathbf{A}_{(m\times n)} \underline{\Delta}\mathbf{G}_{(n\times 1)} \tag{28}$$

When $m > n$, the system may be overdetermined, and the most probable matrix (also called the n-dimensional solution vector) $\underline{\Delta}\mathbf{G}$ is defined to be the one that minimizes the Euclidean length (sum of squares of the components) of the m-dimensional vector $\mathbf{Q}$. This is equivalent to minimizing the value of $\mathbf{Q}^T\mathbf{W}\mathbf{Q}$ where $\mathbf{Q}^T$ denotes the transpose of matrix $\mathbf{Q}$. The solution vector $\underline{\Delta}\mathbf{G}$ is known to satisfy

$$\mathbf{A}^T\mathbf{W}\mathbf{A}\,\underline{\Delta}\mathbf{G} = \mathbf{A}^T\mathbf{W}\mathbf{Q} \tag{29}$$

from which

$$\underline{\Delta}\mathbf{G} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{Q} \tag{30}$$

if the inverse $(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}$ exists. The matrix $\mathbf{A}^T\mathbf{W}\mathbf{A}$ is the weighted normal matrix; Eq. (29) represents the so-called normal equations.

If $\boldsymbol{\varepsilon}$ represents the m-dimensional error vector of the observations, the error in $\underline{\Delta}\mathbf{G}$ due to $\boldsymbol{\varepsilon}$ is then

$$\delta\underline{\Delta}\mathbf{G} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\boldsymbol{\varepsilon} \tag{31}$$

This must be distinguished from the error

$$\delta\Delta\mathbf{G} = \mathbf{G} - \mathbf{G}' - \underline{\Delta}\mathbf{G} \tag{32}$$

resulting from the attempt to solve for the difference $\Delta\mathbf{G}$ between the most probable ($\mathbf{G}$) and approximate ($\mathbf{G}'$) values of the parameters using the truncated Taylor series. The quantity $\delta\underline{\Delta}\mathbf{G}$ is a measure of the worth of the solution vector $\underline{\Delta}\mathbf{G}$ as determined by the quality of the data used to solve for it. The covariance matrix $\boldsymbol{\Gamma}_x$ on the solution is defined by

$$\begin{aligned} \boldsymbol{\Gamma}_x &= (\delta\underline{\Delta}\mathbf{G})(\delta\underline{\Delta}\mathbf{G})^T \\ &= [(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\boldsymbol{\varepsilon}]\,[(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\boldsymbol{\varepsilon}]^T \\ &= (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\overline{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T}\mathbf{W}^T\mathbf{A}\,(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1T} \end{aligned} \tag{33}$$

where $\overline{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T}$ denotes the value obtained using the average (or most probable) $\boldsymbol{\varepsilon}$ chosen from the infinite set of possible error vectors $\boldsymbol{\varepsilon}_i$. The quantity $\overline{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T}$ is the covariance matrix of the data $\boldsymbol{\Gamma}_D$, and is generally unknown. The common practice is to assume

$$\overline{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T} = \boldsymbol{\Gamma}_D \simeq \mathbf{W}^{-1} \tag{34}$$

where $\mathbf{W}$ is a positive definite symmetric weighting matrix, in which case Eq. (33) reduces to

$$\Gamma_x = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{W}^{-1}\mathbf{W}\mathbf{A}(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}$$
$$= (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1} \qquad (35)$$

If the observations are all independent and equally weighted, with standard deviation σ, then $\mathbf{W} = \mathbf{I}/\sigma^2$, where $\mathbf{I}$ is the identity matrix, and

$$\Gamma_x = \sigma^2(\mathbf{A}^T\mathbf{A})^{-1} \qquad (36)$$

Correlation coefficients are found from Γ_x by dividing each row and column by the square root of its component on the major diagonal of Γ_x.

The standard deviation σ_u of an observation of unit weight is usually approximated by

$$\sigma_u^2 \simeq (\mathbf{Q} - \mathbf{A}\underline{\Delta}\mathbf{G})^T\frac{(\mathbf{Q} - \mathbf{A}\underline{\Delta}\mathbf{G})}{m-n} \qquad (37)$$

from which the probable error λ_u of an observation of unit weight is expressed by

$$\lambda_u = 0.6745\,\sigma_u \qquad (38)$$

The probable error λ_k of the quantity $\underline{\Delta}g_k$ is given in terms of the probable error of an observation of unit weight by

$$\lambda_k = \lambda_u\Gamma_{x_{kk}}^{1/2} \qquad (39)$$

The value of $\underline{\Delta}\mathbf{G}$ obtained from Eq. (30) is used to correct the parameter vector $\mathbf{G}'$, which can then be employed to compute new values of the quantities F'_t. The process is repeated until the Euclidean length of $\mathbf{Q}$ converges.

The actual observed quantities $\tilde{f}_t$ are the angles α and δ. Their functional expressions are given in Eqs. (40) and (41).

V. Differential Correction Coefficients

The coefficients in the conditional equations used for differential correction are the partial derivatives of functions of the computed observable with respect to the parameters whose values are to be improved. In each equation, the empirical term is related to the difference between the observed and computed values of a quantity. The formation of a conditional equation corresponding to a particular observation therefore involves two separate processes: (1) obtaining a computed value for the observable and (2) evaluating partial derivatives of that computed quantity with respect to the necessary parameters.

The right ascension and declination of a body are related to the rectangular coordinates $\mathbf{r}_*$ of the observed object and $\mathbf{r}_\phi$ of the observer by

$$\alpha_c = \arctan\frac{\rho_y}{\rho_x} \qquad (40)$$

$$\delta_c = \arctan\frac{\rho_z}{(\rho_x^2 + \rho_y^2)^{1/2}} \qquad (41)$$

where

$$\boldsymbol{\rho}(t, t') = \mathbf{r}_*(t') - \mathbf{r}_\phi(t) = (\rho_x, \rho_y, \rho_z) \qquad (42)$$

The quantities t' and t represent respectively the time at which light left the object and the instant at which the observer saw the light, namely, the time of the observation. The subscript c stresses that Eqs. (40) and (41) represent computed observables. It is assumed that (α, δ), $\mathbf{r}_*$, and $\mathbf{r}_\phi$ are referred to the same equator and equinox. For higher accuracy, when the arguments in Eqs. (40) and (41) are greater than unity, the angles should be calculated from the arc cotangent of the reciprocal arguments.

The use of t' in Eq. (42) accounts for the portion of planetary aberration known as the correction for light time. The remaining component, stellar aberration (diurnal and annual), is discussed in Section VI.

The light time $t - t'$ in days satisfies the condition that

$$t - t' = \frac{|\boldsymbol{\rho}(t, t')|}{\mathbf{C}_A} \qquad (43)$$

where $\mathbf{C}_A$ is the speed of light in AU/day. The value of t' is calculated to a precision of 10^{-6} days by iterative solution of Eq. (43), starting with $\mathbf{r}_*(t)$ and continuing thereafter with $\mathbf{r}_*(t')$. It is essential to note that the positions, velocities, and partial derivatives of the observed object, for whatever use in differential correction, must be those for time t', whereas the corresponding quantities for the observer refer to time t.

The value of $\mathbf{r}_\phi(t)$ is obtained from the position of the center of the earth $\mathbf{r}_1(t)$, in the frame of reference being used, and the geocentric position of the observer $\mathbf{r}_2(t)$ described in Eqs. (76) through (80) by

$$\mathbf{r}_\phi(t) = \mathbf{r}_1(t) + \mathbf{r}_2(t) \tag{44}$$

The heliocentric coordinates of the earth can be obtained in a number of ways. The most common approach has been to interpolate from Ref. 12. An alternate method (and the one used here) is the evaluation of Newcomb's theory of the sun (see Ref. 32) for the instant of the observation. The modifications used in this work to bring the theory of the sun into closer coincidence with Ref. 12, which is based upon the Tables of the Sun, are presented in Appendix A. A more consistent approach would be to take the position of the earth from the perturbation ephemeris that is used for generating the orbit of the object being observed. If the ephemeris contained the earth–moon barycenter, the heliocentric position of the earth could be derived using a simplified lunar theory described by Lieske (see Ref. 9) or Fiala (Ref. 13).

The computation of partial derivatives can also be separated conceptually into two parts. Since α and δ may be defined in terms of the rectangular coordinates, it is possible first to express the derivatives of the angles with respect to these quantities, and then to combine the results with the derivatives of the rectangular coordinates with respect to the desired n parameters p_i. Thus,

$$\begin{bmatrix} \alpha_0 - \alpha_c \\ \\ \delta_0 - \delta_c \end{bmatrix} = \begin{bmatrix} \dfrac{\partial(\alpha,\delta)_c}{\partial(p_1, \cdots p_n)} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \vdots \\ \Delta p_n \end{bmatrix}$$

$$= \left[\frac{\partial(\alpha,\delta)_c}{\partial(x,y,z)_*}\right] \left[\frac{\partial(x,y,z)_*}{\partial(p_1, \cdots p_n)}\right]$$

$$+ \left[\frac{\partial(\alpha,\delta)_c}{\partial(x,y,z)_\phi}\right] \left[\frac{\partial(x,y,z)_\phi}{\partial(p_1, \cdots p_n)}\right] \tag{45}$$

where $(\alpha,\delta)_0$ are the values observed at the time t, for which Eqs. (40) and (41) yield $(\alpha,\delta)_c$.

The first quantity of each term in Eq. (45) is

$$\left[\frac{\partial(\alpha,\delta)_c}{\partial(x,y,z)}\right]^*_\phi =$$

$$\begin{matrix} + \\ - \end{matrix} \begin{bmatrix} \dfrac{-\rho_y}{\rho_x^2 + \rho_y^2} & \dfrac{\rho_x}{\rho_x^2 + \rho_y^2} & 0 \\ \dfrac{-\rho_x\rho_z}{\rho^2(\rho_x^2 + \rho_y^2)^{1/2}} & \dfrac{-\rho_y\rho_z}{\rho^2(\rho_x^2 + \rho_y^2)^{1/2}} & \dfrac{(\rho_x^2 + \rho_y^2)^{1/2}}{\rho^2} \end{bmatrix}$$

$$= \pm\frac{1}{\rho} \begin{bmatrix} \dfrac{-\sin\alpha}{\cos\delta} & \dfrac{\cos\alpha}{\cos\delta} & 0 \\ -\cos\alpha\sin\delta & -\sin\alpha\sin\delta & \cos\delta \end{bmatrix} \tag{46}$$

The unknown quantities to be obtained in this report are corrections to the orbital parameters of (48) Doris and a correction to the mass of Jupiter. The restriction to this set of quantities is covered in Section VIII. Partial derivatives for each of the unknowns are discussed in turn.

The orbit can be specified by numerous sets of parameters. This report makes use of two commonly used sets: (1) the epoch state vector of rectangular coordinates and velocities ($\mathbf{r}_0$, $\dot{\mathbf{r}}_0$) and (2) the elements of the ellipse osculating at epoch. As was seen earlier, the most direct and conceptually the simplest method of correcting an integrated orbit is to adjust the initial state vector. Correcting the ecliptic Keplerian elements a, e, i, Ω, ω, and M_0 has the advantage of facilitating a feeling for the magnitude and effect of orbital changes. Because either set of elements can be expressed in terms of the other, the correction techniques are theoretically equivalent, although they may not give identical results in practice. A discussion follows of some methods for obtaining partial derivatives of the instantaneous rectangular coordinates with respect to both of these sets of parameters.

From a theoretical point of view, the partial derivatives most valuable for correcting the initial state vector directly are the variational equations defined by Eq. (5). If these are integrated as written, their precision would be limited by the integration order and stepsize, and the computer word length and roundoff. This was the primary approach in the investigation, and the results are presented and compared with other methods in Section VIII.

A commonly employed substitute for this exact technique is that of finite-difference partial derivatives. The mean value theorem of calculus implies that the derivative of a function at some point can be approximated by the slope of a chord connecting adjacent points on either side of the nominal value. The accuracy of the approximation depends upon the choice of adjacent points. If some parameter p_{i_0} upon which the functions x,y,z depend is perturbed by an arbitrary Δp_i, then—from the above considerations and from the definition of a derivative—it is seen that

$$\left.\frac{\partial(x,y,z)}{\partial p_i}\right|_{p_{i_0}} \simeq \frac{\Delta(x,y,z)}{2\Delta p_i} = \frac{(x,y,z)_{p_{i_0}+\Delta p_i} - (x,y,z)_{p_{i_0}-\Delta p_i}}{2\Delta p_i} \tag{47}$$

Because the numerical integration of x,y,z requires a large amount of computation, often only one perturbed value is computed, and Eq. (47) is approximated by

$$\frac{\partial(x,y,z)}{\partial p_i} \simeq \frac{\Delta(x,y,z)}{\Delta p_i} = \frac{(x,y,z)_{p_{i_0}+\Delta p_i} - (x,y,z)_{p_{i_0}}}{\Delta p_i} \tag{48}$$

The degree to which Eq. (48) is satisfied depends on the size of Δp_i. By the definition of the derivative, Δp_i should be very small, but since the two values of x,y,z will be very close to one another and the word length of any computer is finite, the difference between the values of x,y,z will be much less significant than the values of x,y,z themselves, and the derived quantities will represent the actual derivatives only to the number of digits in the difference. On the other hand, if Δp_i is large, and x,y,z change rapidly with p_i, the derivative obtained from Eq. (48) has less chance of agreeing with the actual derivative than that derived from Eq. (47), since it is equivalent to the actual derivative at some point between p_{i_0} and $p_{i_0} + \Delta p_i$. The results for the parameter improvements using this technique are presented in Section VIII.

A very economical approach to differential correction, applicable even to manual calculation, is the widely used method of Eckert and Brouwer (Ref. 14). For orbits that are not highly perturbed, the perturbed state vector at time t can be closely approximated by the state vector at that time on the ellipse osculating at epoch. Because the derivatives of the Keplerian state vector with respect to the osculating elements are easily evaluated, the initial state vector can be corrected through corrections to the elements. The quantities solved for, $\Delta\boldsymbol{\xi}$ (described in Appendix B), are six functions of the elements and the corrections to them. The analytical forms of the partial derivatives $\mathbf{D}(t)$, where

$$\mathbf{D}(t) = \frac{\partial(\mathrm{x, y, z}, \dot{\mathrm{x}}, \dot{\mathrm{y}}, \dot{\mathrm{z}})}{\partial(\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6)} \tag{49}$$

are given in Appendix B. Corrections to the osculating elements can be obtained from the expressions for the quantities $\Delta\xi_i$. The changes in the Keplerian state vector at any time t_i are derived in terms of the solution vector $\Delta\boldsymbol{\xi}$ by

$$\begin{bmatrix} \Delta \mathrm{x} \\ \Delta \mathrm{y} \\ \cdot \\ \cdot \\ \Delta \mathrm{z} \end{bmatrix}_{t_i} = \mathbf{D}(t_i) \begin{bmatrix} \Delta\xi_1 \\ \Delta\xi_2 \\ \cdot \\ \cdot \\ \Delta\xi_6 \end{bmatrix} \tag{50}$$

The corrections at $t = t_0$, because of the definition of an osculating orbit, will represent corrections to the initial state vector for the perturbed orbit.

The three basic sets of unknowns proposed by Clemence and Brouwer in chapter 9 of Ref. 2, are attempts at economizing the labor involved in computing the derivatives when some forehand knowledge about the orbit itself is available. The most generalized form (set 3) is commonly used in differential correction computer programs. The probable errors arising from solutions using each set are discussed in Section VIII. Other individuals have published their own preferred sets of parameters, but the sets mentioned above are especially well known.

The implementation of these elliptic approximation schemes depends upon the understanding an individual has of the philosophy behind using the derivatives of Eq. (49) and the degree of his desire for economy of computation.

Let $\partial\mathbf{U}/\partial\boldsymbol{\xi}$ represent the matrix of analytical expressions for the partial derivatives of the state vector of rectangular coordinates and velocities $\mathbf{U}$ with respect to the parameters $\boldsymbol{\xi}$. When osculating elements $\mathbf{E}_i$ and a state vector $\mathbf{U}_j$ are used in these expressions, the matrix of evaluated derivatives is denoted $\partial\mathbf{U}/\partial\boldsymbol{\xi}\ (\mathbf{E}_i, \mathbf{U}_j)$. If the perturbed state vector and the Keplerian state vector at time t are represented by $\mathbf{U}_p$ and $\mathbf{U}_k$ respectively, and the elements of the ellipses osculating at t and epoch are represented by $\mathbf{E}_t$ and $\mathbf{E}_0$, then some common ways in which Eq. (49) has been interpreted are

$$\mathbf{D}_1(t) = \frac{\partial\mathbf{U}}{\partial\boldsymbol{\xi}}\,(\mathbf{E}_0, \mathbf{U}_k) \tag{51}$$

$$\mathbf{D}_2(t) = \frac{\partial\mathbf{U}}{\partial\boldsymbol{\xi}}\,(\mathbf{E}_0, \mathbf{U}_p) \tag{52}$$

$$\mathbf{D}_3(t) = \frac{\partial\mathbf{U}}{\partial\boldsymbol{\xi}}\,(\mathbf{E}_t, \mathbf{U}_p)\,\frac{\partial\boldsymbol{\xi}(\mathbf{E}_t)}{\partial\boldsymbol{\xi}(\mathbf{E}_0)} \tag{53}$$

One way of choosing between these forms is to argue that, since the idea is to correct an elliptic approximation to the true orbit, only state vectors from that elliptic orbit should be used to compute $\mathbf{D}(t)$ (Eq. 51). This involves computing $\mathbf{U}_k$, however, which might be approximated by the $\mathbf{U}_p$ already available from the integration, as Eckert and Brouwer (see Ref. 14) imply (Eq. 52). The use of Eq. (53) for differential correction involves no approximation if the second term is obtained from a variation-of-elements technique. The assumption is sometimes made, however, that this term can be represented adequately by the identity matrix, implying that

$$\frac{\partial \mathbf{U}}{\partial \boldsymbol{\xi}}(\mathbf{E}_t, \mathbf{U}_p) \simeq \frac{\partial \mathbf{U}}{\partial \boldsymbol{\xi}}(\mathbf{E}_0, \mathbf{U}_p) \tag{54}$$

Cohen, Hubbard, and Oesterwinter (Ref. 15) experimented with this approach on the orbit of Pluto and realized very slow convergence to values far from those obtained using Eq. (51). They concluded that part of their problem was the assumption that $\mathbf{G} = \partial \boldsymbol{\xi}(\mathbf{E}_t)/\partial \boldsymbol{\xi}(\mathbf{E}_0) = \mathbf{I}$. The results of using Eq. (51) in each of the three Eckert–Brouwer sets are described in Section VIII.

The coefficients for correction of the mass can take either of two forms, differing by a factor of m_j in the expression for the partial derivative $\partial \ddot{\mathbf{r}}_i/\partial$ (mass) in Eq. (4). The correction to the mass may be viewed as an increment Δm or a multiplicative factor θ. The factor approach $(\partial \mathbf{r}_i/\partial \theta_j)$ was chosen because it permits m_j to remain in the term mentioned above. This restricts the magnitude of the derivative itself, enhancing the accuracy of the integration. The choice of solving for the increment Δm using $\partial \mathbf{r}_i/\partial m_j$ merely involves removing m_j from the expression in question.

VI. Reduction of Observations

Classical astronomical observations consist of angular measurements of the position of an object, as well as the time at which the measurements are made. This section covers the reduction of both types of data to a common system so that they can be more readily used to compare with a computed orbit. Also, this section treats the effect of observatory location on the observation.

A. Observed Angles

The published coordinates of an object are either mean or apparent places. A true mean place consists of right ascension α and declination δ with respect to some mean equator and equinox. An apparent place consists of coordinates referred to the true equator and equinox of date and modified by annual aberration. The mean place referred to in this section, unless otherwise indicated, denotes the true mean place augmented by the elliptic terms of annual aberration at that α and δ. Stellar catalogs by convention contain mean places of stars in this second sense. Consequently, the transformation procedures from true mean to apparent place have been modified to apply to catalog mean places, and it is these that are commonly found in astronomy today.

The investigator who wishes to compare observations with an orbit on some fixed equator and equinox can either compute apparent places from the orbit or reduce the observations to true mean positions. The second approach (the one taken herein) requires that apparent observations be reduced to mean places. All observations must then be transformed to true mean places on the fixed equator and equinox of the orbit.

Coordinate observations are of three basic types: photographic, visual-micrometric, and visual-transit. Each requires a different procedure for reduction to mean place.

Photographic positions are based upon differences between the plate coordinates of an object and three or more reference stars. The determination of the equatorial coordinates for the body involves the standard coordinates of the mean places of the reference stars at the instant of observation. These stellar positions should include proper motion, but few observatories document their published observations sufficiently to indicate whether or not this is the case. Because the star positions used are all mean places on some arbitrary equator and equinox (usually that of the beginning of some Besselian year), the derived positions will also be expressed in mean coordinates, on the same equator and equinox as the stars.

The various plate reduction techniques account for first-order differential refraction, aberration, precession, and nutation, which affect the apparent positions of objects on the plate. Second-order effects are usually negligible compared with the precision of measurement of the images.

Visual-micrometer observations consist of the apparent angular separation $\Delta\alpha, \Delta\delta$ between an object and a reference star. The actual separation can be obtained by eliminating the differential effects mentioned above. It is then possible to obtain the position of the object in terms of that of the reference star. The prevalent custom among visual-micrometer users is to express apparent positions for the objects they observe. To do so, they must compute an apparent place for the reference star from a catalog, add the observed $\Delta\alpha,\Delta\delta$, and account for the differential refraction. A few observatories publish the $\Delta\rho$ (refraction) and all of the raw data for computing the $\Delta\alpha,\Delta\delta$, but what usually appears is just a mean place for the reference star, the $\Delta\alpha,\Delta\delta$, and the deduced apparent place of the object.

To eliminate accidental errors and to systematize reductions, as well as to employ presumably better-known modern positions and proper motions of reference stars, micrometric observations were rereduced whenever possible. This was done by computing an apparent place for the reference star at the time of observation using modern positions, proper motions, and transformation techniques; adding the $\Delta\alpha$ and $\Delta\delta$; and using the resultant apparent place as a corrected position. The merits of this approach are covered in Section VII.

The reference stars were located in the *Geschichte des Fixsternhimmels* (Ref. 16), and identified by Bonner Durchmusterung (BD) number. These numbers were used to search the Yale catalogs (Ref. 17) for relatively modern positions. Most of the northern stars not in the Yale catalogs were found in AGK2 (Ref. 18). When proper motions were available, they were applied from the epoch of observation of the star to the epoch of observation of (48) Doris. The corresponding position on the equator and equinox of 1950.0 was then precessed to the beginning of a solar year nearest to the date of observation for use in the apparent place reductions.

Meridian-transit observations are more direct measurements of position than the micrometric type in that a calibrated observing system is maintained for giving the apparent coordinates, basically in terms of the time and zenith distance of meridian transit.

The transformations from mean to apparent and vice versa involve the coordinates of the object and various constants (day numbers) related to the amounts of precession, nutation, and aberration affecting observations each day. The following is a discussion of the transformation methods and the derivations of the day numbers used in the reductions.

The fact that detailed expressions are available only for transformations from mean to apparent place, and that these formulas are not truly reversible, has led to the introduction of a number of approximations to convert from apparent to mean. The most common approach is a single evaluation of the correction, apparent − mean, using the apparent place in lieu of the mean place in the equations. The computed correction is then applied, with the opposite sign, to the apparent place to derive an approximate mean place. This derived mean place can be substituted in the expressions for apparent − mean and the result compared with the original apparent place to differentially correct the mean place. The apparent − mean corrections in this report involve the second-order expressions in Woolard and Clemence (Ref. 19, p. 319).

The experience of positional astronomers is that, when mean places are referred to the beginnings of Besselian solar years, the most accurate and efficient application of the Besselian day numbers is in computing apparent places from places referred to the nearest beginning of a solar year.

The day numbers were computed directly for the instant of observation. Evaluation of the nutation in longitude $\Delta\psi$ and the nutation in obliquity $\Delta\epsilon$ from Woolard's theory of nutation (Ref. 20) enables one to obtain A, B, and E from

$$A = \left(\tau + \frac{\Delta\psi}{\psi'}\right)\psi' \sin\epsilon' \tag{55}$$

$$B = -\Delta\epsilon \tag{56}$$

$$E = \lambda' \frac{\Delta\psi}{\psi'} \tag{57}$$

Here τ denotes the fraction of a tropical year of 365.241988 mean solar days from the beginning of the nearest Besselian solar year to date; ψ' is the annual rate of lunisolar precession on the fixed ecliptic of date; λ' is the annual rate of planetary precession at date; ϵ' is the mean obliquity of date (differing from the true obliquity of date ϵ by $\Delta\epsilon$).

The aberrational day numbers for true mean to apparent reductions are obtained from the ecliptic solar system barycentric velocity x', y', z' of the earth by the expressions

$$C' = \frac{y'}{c'} \tag{58}$$

$$D' = \frac{-x'}{c'} \tag{59}$$

If the velocities are in AU/day, the denominator c' is given by $C_A \sin 1''$, where C_A is the velocity of light in AU/day. The velocities, if not otherwise available, can be computed by numerically differentiating positions. Barycentric positions of the earth can be obtained by combining heliocentric coordinates of the earth with barycentric coordinates of the sun, which can be derived to the required accuracy by c.m. considerations from elliptic orbits of Jupiter, Saturn, Uranus, and Neptune. The velocities are customarily referred to the equator and equinox of the nearest beginning of a Besselian year.

If the heliocentric position of the earth is derived from Newcomb's theory of the sun, modification of the terms expressing the lunar perturbations is advisable. This would account for the effects of the improved value of the earth–moon mass ratio upon the coordinates of the earth with respect to the barycenter.

The reduction from catalog mean place to apparent place differs from the reduction from true mean place by the elliptic portion of annual aberration. In terms of the eccentricity e and longitude of perihelion ϖ of the earth's orbit evaluated at the nearest beginning of a Besselian year and the aberrational constant k, the catalog aberrational day numbers are expressed in terms of the true mean quantities of Eqs. (58) and (59) as

$$C = C' + \Delta C = C' - ke \cos \varpi \cos \epsilon' \tag{60}$$

$$D = D' + \Delta D = D' - ke \sin \varpi \tag{61}$$

Diurnal aberration (apparent − mean) is computed by the expressions

$$\Delta\alpha = 0''.0213\, \rho \cos \phi' \cos H \sec \delta \tag{62}$$

$$\Delta\delta = 0''.3200\, \rho \cos \phi' \sin H \sec \delta \tag{63}$$

where ϕ' is the observer's latitude, and H and ρ are given by Eqs. (73)–(75) and (79).

The computer programs designed for computation of the day numbers and the reduction from mean to apparent place produce results agreeing to 0″.001 with programs currently used at the United States Naval Observatory (USNO).

To reduce a true mean place α, δ at one time t to α_0, δ_0 at another time t_0, the Newcomb precession constants

$$\zeta_0 = (2304''.250 + 1''.396 T_0) T + 0''.302 T^2 + 0''.018 T^3 \tag{64}$$

$$z = \zeta_0 + 0''.791 T^2 \tag{65}$$

$$\theta = (2004''.682 - 0''.853 T_0) T - 0''.426 T^2 - 0''.042 T^3 \tag{66}$$

are used in the formulas

$$\cos \delta \sin (\alpha - z) = \cos \delta_0 \sin (\alpha_0 + \zeta_0) \tag{67}$$

$$\cos \delta \cos (\alpha - z) = \cos \theta \cos \delta_0 \cos (\alpha_0 + \zeta_0) - \sin \theta \sin \delta_0 \tag{68}$$

$$\sin \delta = \cos \theta \sin \delta_0 + \sin \theta \cos \delta_0 \cos (\alpha_0 + \zeta_0) \tag{69}$$

If t and t_0 are Julian Ephemeris Dates (JED), then the T and T_0 of Eqs. (64)–(66), given as tropical centuries, are defined in terms of the date of 1900.0 (JED 2415020.814) by

$$T_0 = (t_0 - 2415020.814)/36524.1988 \tag{70}$$

$$T = (t - t_0)/36524.1988 \tag{71}$$

The JED of the beginning of any Besselian year is given by

$$\text{JED} = 2415020.814 + 365.241988\,(\text{year} - 1900) \tag{72}$$

The reference equinox for the computed orbit was that of 1950.0.

B. Observation Time

Times of observation are given in five forms:

(1) The UT in hours, minutes, and seconds.

(2) The local mean time (differing from UT by the longitude of the observatory).

(3) The fraction of a mean solar day since 0^hUT.

(4) The local sidereal time of the observatory.

(5) The day of the observation (for some meridian circles).

The times are reduced to form 3 and added to the Julian date at 0^hUT. For forms 1 and 2, there is often some ambiguity as to whether an observation made before 1925 has been corrected by the necessary 0.5 day (Ref. 21). A helpful check is to examine the hour angle

H, derived from the sidereal time S and the observed right ascension α, by the relation

$$H = S - \alpha \tag{73}$$

For this purpose, it is immaterial whether the true or mean sidereal time is used. For subsequent use in determining the UT of an observation, however, the distinction must be made.

The true sidereal time S differs from the mean sidereal time S' at the same instant by the equation of the equinoxes, with

$$S = S' + \Delta\psi \cos \epsilon \tag{74}$$

where $\Delta\psi$ and ϵ are as defined above. In terms of the longitude λ of the observatory, the fraction T_u of a Julian century of 36525 mean solar days from 1900 January 0.5 UT to the beginning of the day of observation, and the fraction γ of a mean solar day since 0^hUT,

$$S' = 6^h 38^m 45^s.836 + 8640184^s.542 T_u + 0^s.0927 T_u^2 + 1.002737909265\gamma - \lambda \tag{75}$$

Form 4 can be reduced to form 3 by use of Eq. (75). When no exact time is given (as in form 5), one assumes that the observed right ascension is the true sidereal time. To reduce from S to S', the true sidereal time may be substituted in Eq. (75) to get a UT for computing the required $\Delta\psi$ and $\Delta\epsilon$. The procedure then follows that for form 4.

Because the observations are made in UT measured by a nonuniformly rotating earth, they must be referred to the uniform ephemeris time scale before the orbital position of the earth at the time of observation can be computed. The corrections ΔT for the years 1820–1952 are found in Ref. 22. Values for more recent years appear in Ref. 23.

C. Observatory Location

The location of the observer affects the apparent position of the object in the sky because of parallax. For comparison with actual observations, computed positions are derived by Eqs. (40) and (41). These involve the geocentric equatorial coordinates of the observer, given with respect to the instantaneous vernal equinox as positive x-axis by

$$x = \rho \cos \phi' \cos S \tag{76}$$

$$y = \rho \cos \phi' \sin S \tag{77}$$

$$z = [\rho(1 - e)^2 + h] \sin \phi' \tag{78}$$

where

$$\rho = a\,[1 - (e \sin \phi')^2]^{-1/2} \tag{79}$$

$$e^2 = f(2 - f) \tag{80}$$

The true sidereal time S has already been defined. The latitude ϕ' and altitude h for each of the various observatories appear in Table 3; the equatorial radius of the earth a and the flattening factor f of the international ellipsoid are given in Table 1. These rectangular coordinates are referred to the true equinox of date, and must be reduced to 1950.0.

VII. Discussion of Observations

A. Collection and Selection

Since the discovery of (48) Doris in 1857, at least 754 observations have been made at 62 different observatories. As originally published, only 599 of these observations appeared to have the necessary precision ($0^s.01$ in α, $0''.1$ in δ, 1^m in time) for use in the differential correction, and even some of these were in a form that was quite difficult to use.

Five 1863 meridian-transit observations from Vienna were reduced from raw data according to the precepts in the *Wien Annalen*. The reduction procedure was checked by comparing computed positions for selected stars with positions actually published in the *Annalen*.

Only differential micrometer measurements $\Delta\alpha$, $\Delta\delta$ were available for 31 observations. These were processed by computing an apparent place for the given reference star, adding the $\Delta\alpha$ and $\Delta\delta$, and continuing as described in Section VI. This procedure not only salvaged the 31 observations that were not otherwise reducible, but, when applied to other micrometer observations for which final positions were published, helped to detect transcription and typographical errors.

All of the 155 observations published with less than the required precision were photographic, and an attempt was made to obtain explanations for the imprecision. One reason given is that plate scales of some cameras are not sufficiently large to permit resolution to $0''.1$. In practice, 70 cm was found to be the focal length below

Table 3. Observatories

IAU[a] No.	Location	Altitude h, m	Longitude, h	m	s	Latitude ϕ', °	′	″
793	Albany	70	04	55	07.12	42	39	12.8
8	Algiers	345	−00	12	08.53	36	48	04.8
30	Arcetri	184	−00	45	01.30	43	45	14.4
6	Barcelona	415	−00	08	30.20	41	24	59.3
57	Belgrade (after 1931)	253	−01	22	03.20	44	48	13.2
−1	Berlin (1835–1913)	47	−00	53	34.80	52	30	16.7
16	Besancon	312	−00	23	57.42	47	14	59.8
520	Bonn	62	−00	28	23.18	50	43	45.0
999	Bordeaux	73	00	02	06.60	44	50	07
73	Bucharest	83	01	44	23.20	44	24	49.4
802	Cambridge, Harvard	24	04	44	31.05	42	22	47.6
−2	Collegio Romano	51	−00	49	55.12	41	53	53.6
35	Copenhagen	14	−00	50	18.69	55	41	12.6
95	Crimea	550	−02	16	04.00	44	43	42.0
−3	Durham	107	00	06	19.75	54	46	06.2
136	Engelhardt, Kazan	121	−03	15	15.74	55	50	20.2
760	Goethe Link Observatory	300	05	45	34.86	39	32	57.7
0	Greenwich	47	00	00	00.00	51	28	38.2
29	Hamburg–Bergedorf	41	−00	40	57.74	53	28	46.9
24	Heidelberg, Konigstuhl	567	−00	34	53.13	49	23	55.2
78	Johannesburg	1741	−01	52	07.00	−26	11	14.0
−4	Josephstadt	214	−01	05	27.17	48	12	53.8
58	Konigsberg	24	−01	21	58.97	54	42	50.5
13	Leiden	6	−00	17	56.15	52	09	19.8
534	Leipzig	119	−00	49	33.92	51	20	05.9
39	Lund	34	−00	52	44.97	55	41	51.6
990	Madrid	655	00	14	45.10	40	24	30.0
14	Marseilles (after 1864)	75	−00	21	34.55	43	18	16.3
330	Nanking, Purple Mountain	367	−07	55	17.02	32	03	59.9
20	Nice	376	−00	29	12.10	43	43	17.0
7	Paris	67	−00	09	20.91	48	50	11.0
794	Poughkeepsie, Vassar	61	04	55	35.16	41	41	18.0
84	Pulkovo	75	−02	01	18.57	59	46	18.5
983	San Fernando	30	00	24	49.30	36	27	42.0
804	Santiago	580	04	42	45.09	−33	33	44.2
338	Shanghai, Zo-Se	100	−08	04	44.75	31	05	47.6
94	Simeis	346	−02	15	59.38	44	24	11.6
420	Sydney	44	−10	04	49.19	−35	41	41.1
388	Tokyo, Mitaka	59	−09	18	10.10	35	40	21.4
4	Toulouse	195	−00	05	51.00	43	36	44.1
334	Tsingtao	78	−08	01	16.71	36	04	11.3
22	Turin (Pino Torinese)	618	−00	31	05.95	45	02	16.3
62	Turku	28	−01	28	55.03	60	27	08.7
12	Uccle	105	−00	17	25.97	50	47	55.0
786	U.S. Naval Observatory	86	05	08	15.78	38	55	14.0
15	Utrecht	14	−00	20	31.01	52	05	09.6
−5	Vienna (before 1879)	186	−01	05	31.61	48	12	35.5
45	Vienna (after 1879)	240	−01	05	21.35	48	13	55.1
558	Warsaw	121	−01	24	07.26	52	13	04.6
−6	Washington, National Observatory	31	05	08	12.15	38	53	38.7
28	Würzburg	200	−00	39	44.71	49	47	27.6
754	Yerkes	334	05	54	13.64	42	34	13.4

[a]IAU = International Astronomical Union. (Negative numbers were arbitrarily assigned to observatories that lacked IAU identification number.)

which precise positions could not be obtained. The aperture of the telescope is also a determining factor, since long exposures allow appreciable motion of the asteroid to distort the images. Under these circumstances it is impossible to obtain post-facto improved measurements.

Another explanation offered greater hope. Investigators who found (48) Doris on their plates while studying other objects, or who did not have the time to completely reduce the observations, often published approximate positions. Because accurate positions can still be obtained if the plates are available, 18 observatories were requested to remeasure positions. Nine observatories returned a total of 82 observations and most of the other institutions gave explanations for not sending positions or plates.

A total of 64 full precision observations were deleted: 5 with justification provided by the observers themselves, 17 because of apparent misidentification, and 42 upon the judgement of the author. One criterion for rejection was a residual $(O-C)$ from the final reference orbit in α and δ that exceeded 15″. Often only one coordinate was erroneous, but no attempt was made to salvage the reasonable one. (The residuals given in one coordinate in the listings are from observations made in only one coordinate.)

All 13 photographic observations made at Algiers from 1915 to 1921, published with aberration corrections, were excluded because comparisons with the final orbit seemed to indicate that the published corrections were inconsistently applied to three of the observations. Because there was no initial indication as to whether any of the published positions already included the correction, the problem had to be resolved by inspecting the residuals. Some of the observations seemed to be corrected with the published $\Delta\alpha$, $\Delta\delta$ and others did not; it was decided, therefore, to discard all of the observations rather than to guess at any of them.

The final number of observations used was 617, including 274 photographic, 57 meridian–transit, 257 rereduced, and 29 nonrereducible micrometer positions.

B. Distribution

The distribution of observations in time can be seen in Fig. 3. There is a pronounced gap of 30 yr from 1871 to 1901 during which only three observations were made. Another period of few observations, 1926–1940, was reinforced by the remeasured photographic positions mentioned above. Since the inequality in longitude for (48) Doris has a 72-yr period, it is impossible at present to cover one complete cycle of the perturbation regardless of the span of observations chosen.

Figure 4 shows the distribution of observations in α and δ; Fig. 5 gives the equatorial x and y coordinates of earth and (48) Doris at the observation times. The discontinuities in α and in the distribution of positions along the orbit of (48) Doris result from its synodic period of about 1.2226 yr, causing every tenth opposition to occur at roughly the same place in the orbit. From inspection of the graph, one should not expect any overall seasonal bias on the observations nor declination errors from restriction to a single catalog zone. It should be noted, however, that such an orbit would be prone to $\Delta\alpha_\alpha$ catalog errors.

C. Weighting

All 617 observations used were weighted equally, although a few other schemes were investigated. One suggestion was to base weighting factors on the standard deviations of each data type from the mean of residuals of all data types. However, because the observation types are quite segregated in time, and no one type exists over a sufficient interval to cover the long-period fluctuation in the residuals, such an approach might actually weaken the mass determination by decreasing the effects of the structure expected in the data. Deviations from means in a series of time blocks might ease the problem of finding suitable weights, but this method immediately raises the question of the size of intervals to be chosen.

Systematic errors or correlations certainly exist between observations made by the same observer and equipment, or using the same reference stars. The determination of these correlations—equivalently the assignment of nonzero values to off-diagonal elements in the data-weighting matrix $\mathbf{W}$—is an extremely arbitrary procedure because very few observatories publish probable errors for their measurements, and even fewer discuss interdependence of observations.

D. Residual Analysis

An analysis of the residuals by observation type and observatory is presented in Table 4.

In Section VI, it was stated that visual (micrometer) observations were specially processed, for it seemed that a systematic reduction of as many micrometer observations as possible would serve to eliminate reduction errors intrinsic to each observatory, and would also take

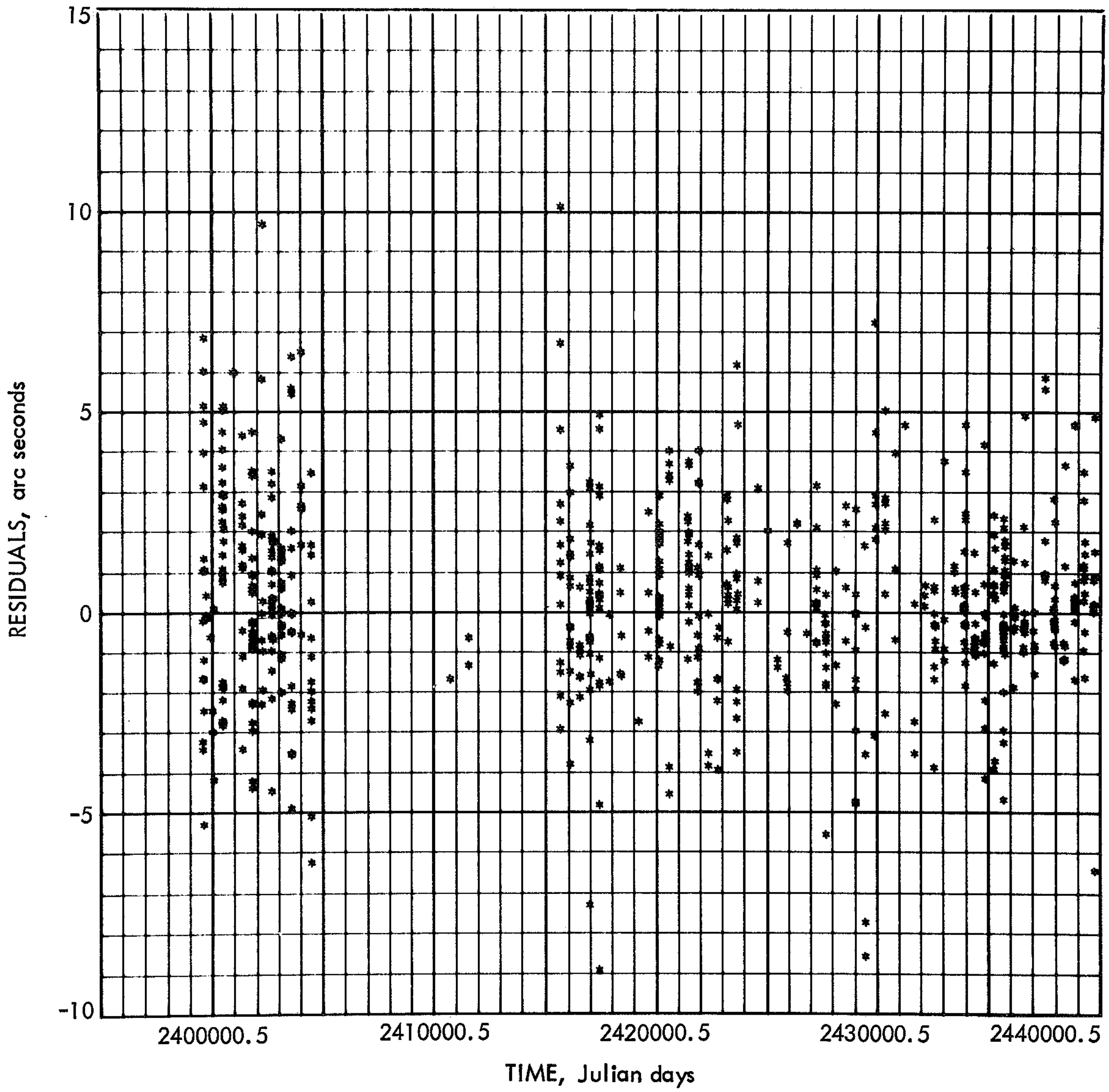

Fig. 3. Right-ascension residuals for reference orbit

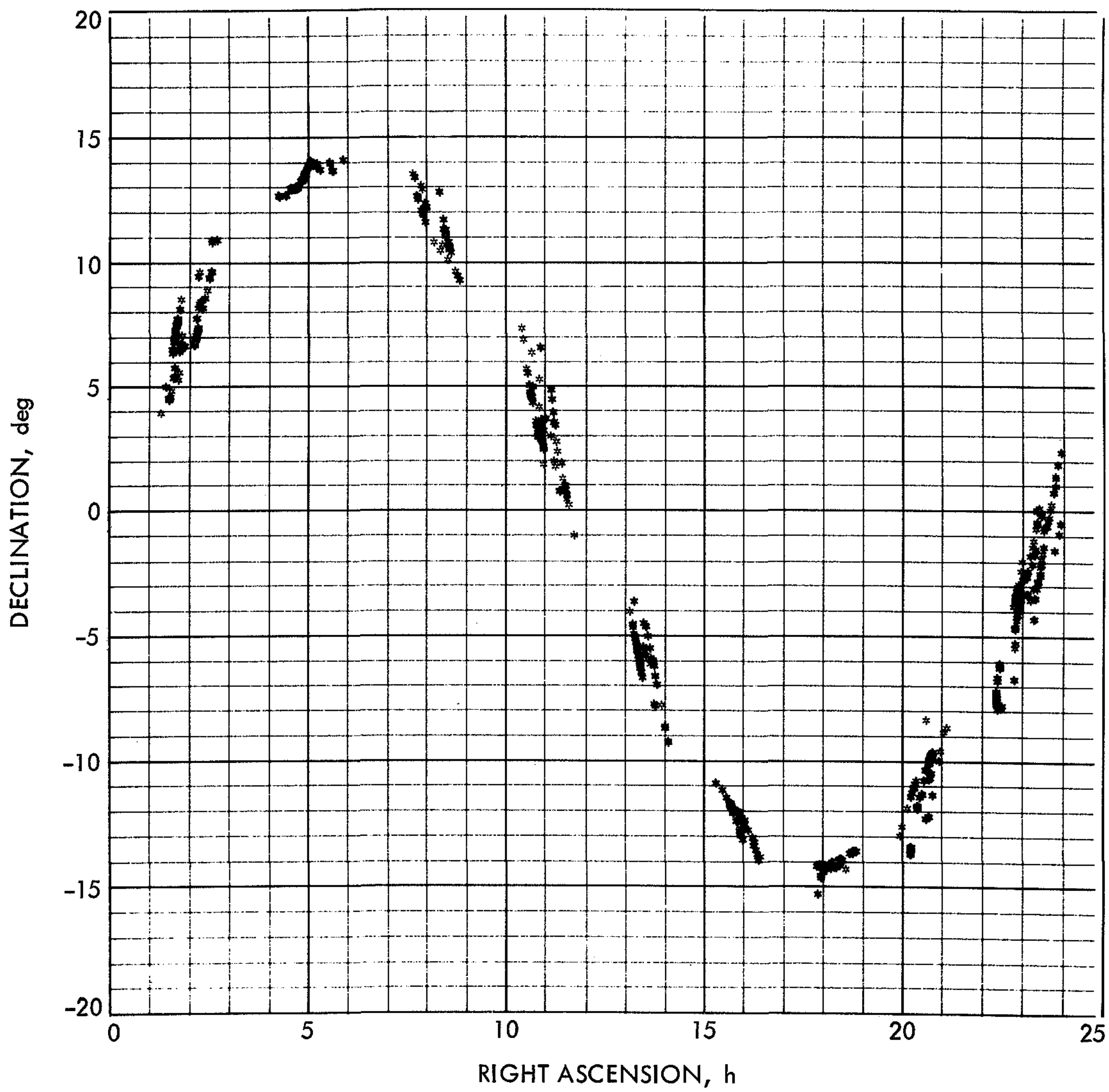

Fig. 4. Distribution of observations

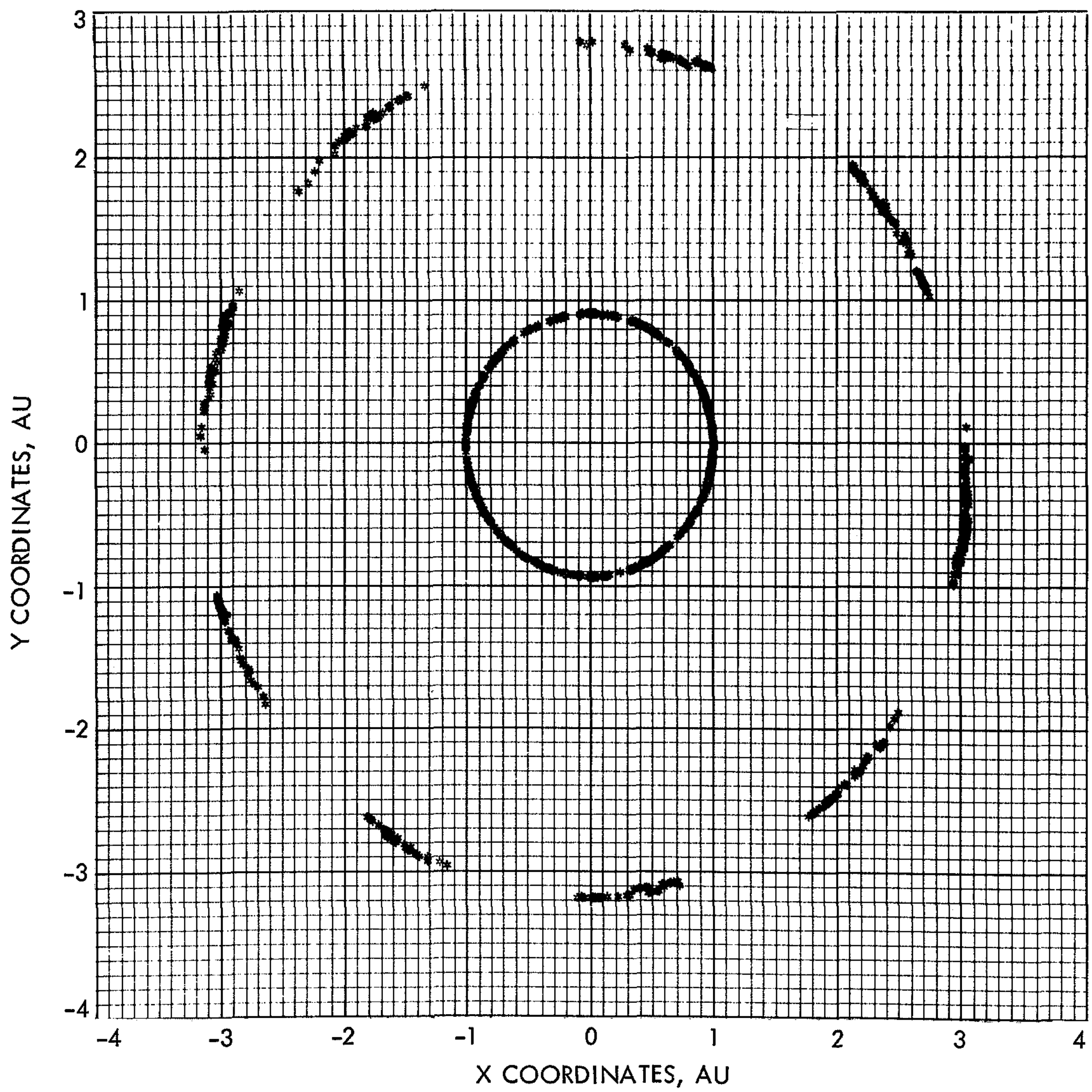

Fig. 5. Equatorial *x* and *y* coordinates of earth and (48) Doris at observation times

Table 4. Analysis of residuals by observatory and type

Observatory	Type of observation												Total		
	Photographic			Meridian			Visual			Rereduced					
	No.	σ_α	σ_δ	No.	σ_α	σ_δ	No.	σ_α	σ_δ	No.	σ_α	σ_δ	No.	σ_α	σ_δ
−6				5	2.147	1.184				8	2.525	2.469	13	2.385	2.239
−5				8	3.027	2.839				15	2.714	3.295	23	2.827	3.144
−4										4	0.689	1.533	4	0.689	1.533
−3										1	5.630	0.367	1	5.630	0.367
−2										2	1.416	1.285	2	1.416	1.285
−1							1	0.107	1.587	34	2.561	1.757	35	2.524	1.752
0				24	3.249	4.197							24	3.249	4.197
4	1	0.513	0.221										1	0.513	0.221
6	3	1.022	1.084										3	1.022	1.084
7				6	2.883	4.885	2	3.327	0.653				8	3.000	4.243
8	10	1.305	0.914							7	4.754	0.948	17	3.211	0.928
12	5	1.537	1.671										5	1.537	1.671
13	22	0.722	0.611	12	1.988	1.470							34	1.334	1.002
14							3	3.590	3.327	5	3.157	1.552	8	3.326	2.331
15										1	0.722	5.430	1	0.722	5.430
16										8	1.380	1.581	8	1.380	1.581
20										6	1.138	1.635	6	1.138	1.635
22	6	2.877	1.671										6	2.877	1.671
24	31	2.557	2.597							1	0.088	2.763	32	2.517	2.602
28	7	5.253	2.625										7	5.253	2.625
29										3	2.587	2.942	3	2.587	2.942
30							7	3.646	1.398	69	1.595	1.749	76	1.880	1.720
35										4	1.622	0.389	4	1.622	0.389
39							1	0.479	2.362	9	3.588	1.040	10	3.407	1.237
45	1	1.365	0.289				9	2.612	1.809				10	2.515	1.718
57	2	0.536	0.651										2	0.536	0.651
58							1	1.082		7	1.538	1.471	8	1.488	1.376
62	5	4.077	1.053										5	4.077	1.053
73	10	1.282	0.756										10	1.282	0.756
78	6	1.415	0.782										6	1.413	0.782
84	1	1.466	1.312				1	3.086	4.939	14	3.082	1.054	16	3.007	1.614
94	1	3.700	1.161										1	3.700	1.161
95	8	1.995	4.037										8	1.995	4.037
136							1	0.466	1.024	14	1.973	1.705	15	1.910	1.668
330	7	2.445	1.238										7	2.445	1.238
334	1	0.670	0.143										1	0.670	0.143
338	3	1.585	0.726										3	1.585	0.726
388	12	2.957	2.133										12	2.957	2.133
420	4	0.833	0.799										4	0.833	0.799
520										4	5.047	2.551	4	5.047	2.551
534							1	6.512	0.159	22	2.381	2.880	23	2.696	2.817
558										4	1.636	1.861	4	1.636	1.861
754	1	0.664	0.679										1	0.664	0.679
760	8	2.186	1.750										8	2.186	1.750
786	81	1.019	0.760										81	1.019	0.760
793				2	1.774	1.266							2	1.774	1.266
794										2	1.231	0.774	2	1.231	0.774
802										1	2.853	2.220	1	2.853	2.220
804							11	3.013	2.114				11	3.013	2.114
983	4	1.042	0.761										4	1.042	0.761
990	34	2.717	1.967										34	2.717	1.967
999										3	1.848	0.342	3	1.848	0.342
Total	274	2.082	1.620	57	2.820	3.444	29	3.255	2.118	257	2.416	1.971	617	2.364	2.024

advantage of modern reference star positions and proper motions. There was some question as to whether the uncertainties in modern proper motions, when propagated over as long a span as 100 yr, would be just as injurious as inaccurate reference star positions taken from old catalogs. A comparison of the residuals in α and δ for 230 micrometer observations appears in Table 5. These residuals were culled from the 257 observations that could be rereduced, and they exclude the aforementioned typographical errors.

Table 5. Comparison of published and rereduced observations

Observations	Standard deviation $(O-C)_\alpha$	Standard deviation $(O-C)_\delta$
Published	$3''.703$	$2''.665$
Rereduced	$2''.279$	$1''.992$

It should be noted that the set of published observations gives a number of residuals over 10″ but less than 15″, which tended to make the improvement with rereduction appear as dramatic as it does.

E. Catalog Corrections

To make the system of observations as homogeneous as possible, some 430 positions were reduced to the FK4 (Ref. 24). Zone corrections were used because there was only one FK4 reference star throughout the 617 observations. Almost all of the rereduced micrometer observations employed stars from the Yale catalog or AGK2. The photographic positions used stars from a number of catalogs, as many as possible of which were reduced to the GC (Ref. 25) and then to the FK4. Only positions were corrected because it was felt that the proper-motion system of the GC was too weak to use as an intermediate reference. The FK4 corrections could be optionally applied during the differential correction process. Since the reference orbit had been fit to uncorrected positions, the sums of squares of residuals $(O-C)$ could increase when the FK4 increments were added. The actual amount of change and its subsequent effect upon the solution parameters are discussed in Section VIII.

VIII. Discussion of Final Results

The determination of the mass of Jupiter from the motion of (48) Doris first required a definitive orbit for the minor planet, based upon the provisional reciprocal mass of 1047.355. Only then could a meaningful investigation of the observations be made for systematic errors before attempting to solve for the correction to the mass.

An orbit determined from Ref. 26 was integrated from its reference epoch (JED 2432200.5) to JED 2440000.5, where rectangular coordinates and osculating elements were extracted. These quantities were used thereafter to describe the orbit, and were differentially corrected using a backward integration over the span from JED 2440000.5 to JED 2399000.5.

The first backward integration was used to compare finite difference and numerically integrated partial derivatives of the rectangular coordinates with respect to the initial rectangular state vector. To form the finite differences, seven bodies were integrated simultaneously under the influence of the sun and nine planets. The first object was (48) Doris, with the above-mentioned rectangular coordinates. Each of the remaining six bodies had either a coordinate perturbed by 10^{-6} AU or a velocity changed by 10^{-8} AU/day. Straightforward differencing and division gave the approximate partial derivatives, which agreed to four digits with the integrated values. Figure 6 displays the numerically integrated $\partial x/\partial x_0$; Fig. 7 shows the difference $\Delta x/\Delta x_0 - \partial x/\partial x_0$.

The orbit was differentially corrected and reintegrated. An attempt was made to improve this orbit, but because the subsequent sum of squares of linearized residuals did not show a marked decrease, this integration was chosen as the reference for subsequent studies with the provisional reciprocal mass 1047.355. Definitive elements for (48) Doris, based upon the reciprocal solar masses in Table 2, appear in Table 6.

The solution parameters were restricted to corrections to the mass of Jupiter and the orbit of (48) Doris. A solution for right-ascension bias, or effect of the equinox correction between FK4 and non-FK4 positions, would be of questionable physical use because all of the observations were not on the same non-FK4 system. Corrections to the orbit of the earth can be accomplished better by observations of other objects, and would only further weaken the solution for the mass. It is possible, without solving for them, to account for the effect of uncertainties in the elements of the orbit of the earth on the solution for the mass, increasing the probable error to a more realistic value. Why this approach was not used is explained below.

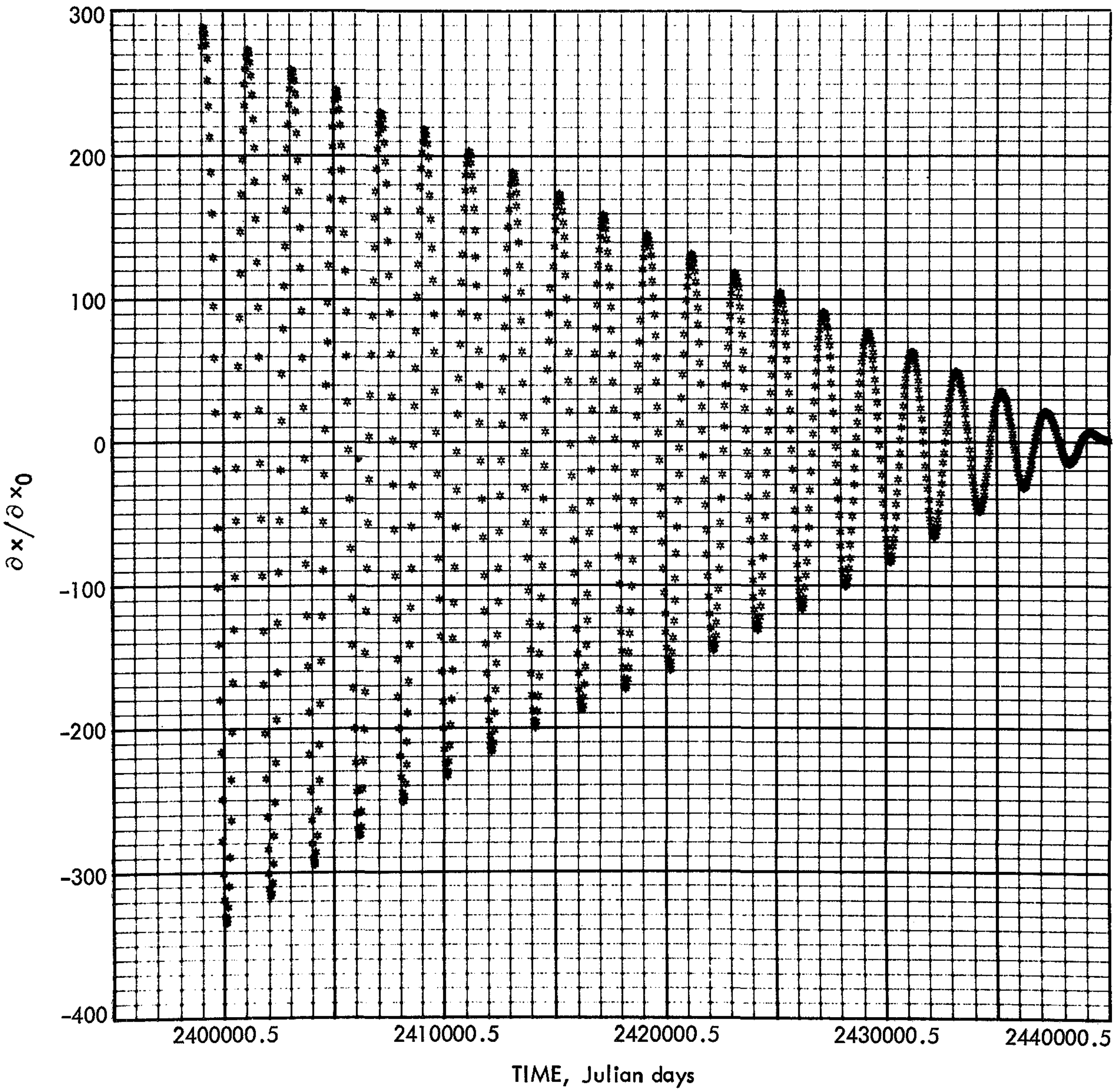

Fig. 6. Numerically integrated partial derivatives $\partial x/\partial x_0$

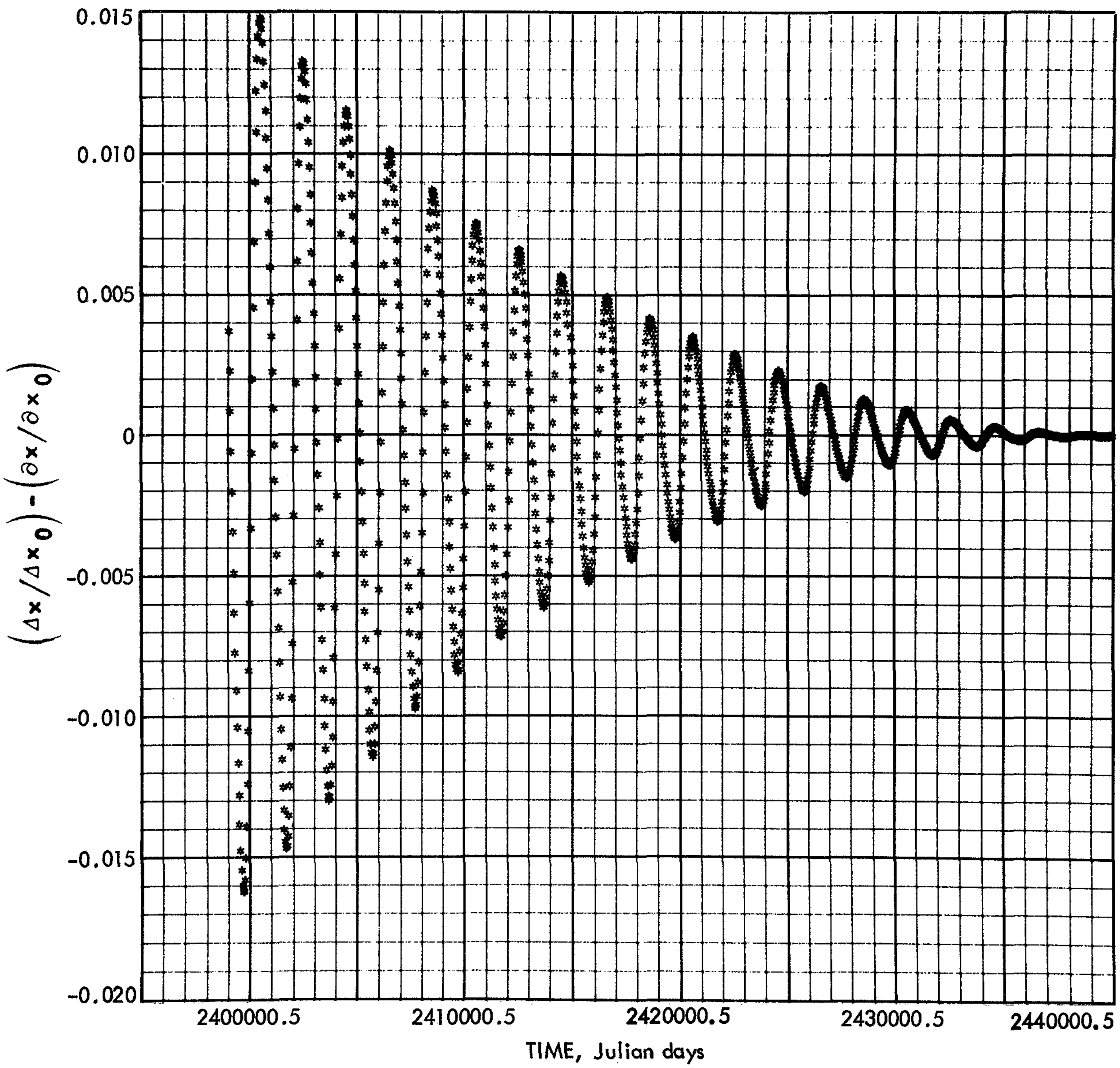

Fig. 7. Finite difference $\Delta x/\Delta x_0$—numerically integrated $\partial x/\partial x_0$

Table 6. Definitive elements for (48) Doris based on the system of masses in Table 1[a]

Symbol	Value	Symbol	Value	Symbol	Value	Symbol	Value
Heliocentric ecliptic Keplerian elements (1950.0)				Equatorial rectangular coordinates (1950.0)			
a	3.1143222812AU	Ω	255.5023183393 deg	x	2.1991122948	$\dot{x}$	−0.007115866686
e	0.0599647307	ω	183.7873456717 deg	y	1.8931264938	$\dot{y}$	0.007057418010
i	6.5476078929 deg	M_0	326.7972322817 deg	z	0.5929347223	$\dot{z}$	0.002089848943

[a]The epoch for both sets of elements is JED 2440000.5, May 24, 1968.

A series of differential corrections was performed for the desired parameters with various sets of unknowns, using the observations shown in Appendix C. The residuals were determined from comparison with this reference orbit, and do not contain any catalog corrections. Graphs of the residuals are shown in Figs. 3 and 8. The following is an analysis of some of these runs, all of which were designed to help indicate the set of parameters that would best determine the mass correction.

Orbit-correction methods were compared first. The three Eckert–Brouwer sets, which were used in solutions for the Keplerian elements, gave identical corrections and probable errors (to 10 significant digits).

The agreement among results using the different methods indicates that the eccentricity of (48) Doris is sufficiently large for the argument of perihelion and the mean anomaly to be well separated. The correlation matrix on the solution for the elements is shown in Table 7. Since it was immaterial which set was used, set 3 became the basis for comparison with the method using variational equations for the rectangular coordinates.

The variational equations reduced the sum of squares of residuals from 5934.0 to 5876.2, whereas the elliptic solutions gave 5877.4. (The units for sums of squares will always be $''^2$.) The corrected rectangular coordinates agreed with those determined from the elliptic approximation to at least 10^{-6} AU in the coordinates and to 10^{-8} AU/day in the velocities. As is shown below, the normal matrix for the variational equations is not as well conditioned as that for set 3; therefore, it would be informative in the future to compare the probable errors of the corrections to the rectangular state vector obtained by both methods. Upon the basis of the studies reported herein, however, both approaches may be considered equally valid for the orbit correction.

The partial derivatives with respect to the mass of Jupiter were numerically integrated in terms of the correction factor θ, as mentioned in Section V. As is shown in Eq. (45), the formal expression for the derivatives of the observed coordinates with respect to the mass of Jupiter involves the derivatives for the earth and for (48) Doris. It was decided, therefore, to compare results obtained with and without the earth terms to justify the contention that they were negligible. Two solutions were made for the mass only, giving a reciprocal mass of 1047.369 ±0.005 in either case. This seemed to indicate beyond a doubt that the earth terms could be neglected.

The final solution for a correction to the mass of Jupiter had to be made simultaneously with an improvement of the orbit of (48) Doris because the orbit is dependent upon the mass. Using the derivatives $\partial \mathbf{r}/\partial \theta$ and the variational equations, the reciprocal mass was determined to be 1047.333 ±0.017. The disparity of this result from those already obtained by O'Handley (Ref. 27), Klepczynski (Ref. 28), and Fiala (see Ref. 13) was initially thought to result from the ill-conditioned normal matrix used for the solution (Table 8). The derivatives $\partial \mathbf{r}/\partial \theta$ were transformed to $\partial \mathbf{r}/\partial m$ by multiplying by 1047.355; the equations were then solved for an increment to the mass of Jupiter, but the results remained unchanged. The elliptic partials were known to give a better-conditioned normal matrix than the variational equations, without arbitrary multiplication of columns and rows; therefore, set 3 was used with $\partial \mathbf{r}/\partial m$, and gave a value of 1047.340 ±0.0156.

The effect of the larger residuals was examined to see whether the results were particularly sensitive to them. Excluding all 50 observations with residuals greater than 5″ (see Figs. 4 and 8) reduced the sum of squares before solution to 3405.2, and gave 1047.344 ±0.014 with the variational equations and 1047.348 ±0.013 with set 3. This proved that the basic solution was not disparate solely because of the few large residuals.

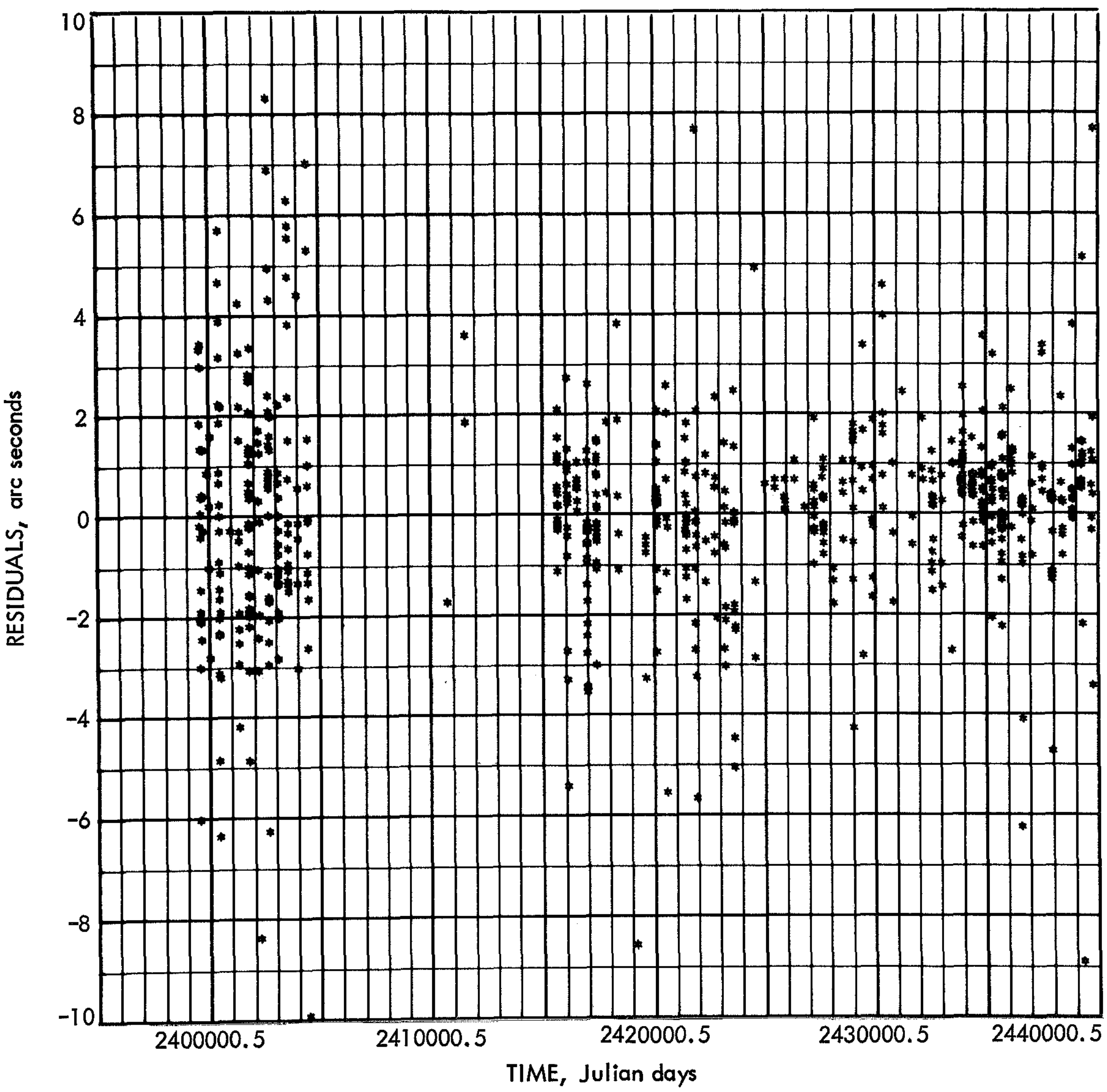

Fig. 8. Declination residuals for reference orbit

Table 7. Correlations in the solution for Keplerian elements

	a	e	I	Ω	ω	M_0
a	0.10000D 01					
e	−0.61700D−01	0.10000D 01				
I	−0.13703D−01	0.20411D−01	0.10000D 01			
Ω	0.18424D−01	−0.28187D−01	−0.58223D 00	0.10000D 01		
ω	−0.12655D−01	0.20626D−01	0.39350D 00	−0.90905D 00	0.10000D 01	
M_0	−0.13118D 00	0.49360D−01	0.66994D 00	−0.87921D 00	0.60897D 00	0.1000D 01

Table 8. Normal matrices for solutions with $\partial \mathbf{r}/\partial\theta$ and variational equations, and with $\partial \mathbf{r}/\partial m$ and Eckert–Brouwer set 3[a]

	$\Delta\theta$	Δx	Δy	Δz	$\Delta\dot{x}$	$\Delta\dot{y}$	$\Delta\dot{z}$	
	0.44320D 01	−0.38118D 04	−0.32911D 04	−0.10283D 04	0.10776D 07	−0.10636D 07	−0.31481D 06	$\Delta\theta$
		0.41529D 07	0.35741D 07	0.11174D 07	−0.11768D 10	0.11575D 10	0.34253D 09	Δx
Δm	0.48617D 07		0.30764D 07	0.96185D 06	−0.10127D 10	0.99634D 09	0.29484D 09	Δy
ξ_1	−0.53450D 05	0.12584D 04		0.30080D 06	−0.31664D 09	0.31151D 09	0.92181D 08	Δz
ξ_2	−0.70815D 04	0.12746D 03	0.73465D 03		0.33347D 12	−0.32800D 12	−0.97063D 11	$\Delta\dot{x}$
ξ_3	0.35723D 04	−0.10345D 03	0.25368D 02	0.54679D 03		0.32269D 12	0.95490D 11	$\Delta\dot{y}$
ξ_4	0.18740D 05	−0.43235D 03	−0.12296D 04	0.72790D 03	0.52654D 04		0.28263D 11	$\Delta\dot{z}$
ξ_5	−0.76615D 07	0.11283D 06	0.11446D 05	−0.92875D 04	−0.40770D 05	0.15118D 08		
ξ_6	0.41627D 05	−0.30287D 03	−0.27033D 02	0.25662D 02	0.17051D 03	−0.19892D 05	0.28967D 04	
	Δm	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5	ξ_6	

[a]Neither case includes the earth terms in the derivatives for the mass. When the earth terms are included, the element $\Delta m,\Delta m$ is 0.52474D 07, from which the modified values of the $\Delta\theta$ row and Δm column can be obtained.

When FK4 corrections were applied, the sum of squares of residuals in δ increased by 23.7, whereas in α it dropped enough to give an overall decrease of 10.1. The derived values for the mass correction and its probable error remained the same.

In view of the fact that the probable error was already so large as to limit the precision of the mass determination to five digits, the inclusion of the effect of uncertainties in the orbit of the earth became more a subject of academic interest than one of basic physical importance, and was not undertaken.

As an independent check on the partial derivatives for the mass used in the simultaneous solution, three orbits were generated, using distinct values of the mass of Jupiter and the same basic set of elements. Each of these orbits was corrected, using set 3 partial derivatives; a parabola was passed through the sums of squares of residuals, and differentiated with respect to the mass. The minimum occurred at 1047.340.

Appendix A

Modifications to Newcomb's Theory of the Sun

In 1948, Clemence (Ref. 29) suggested that the tabular centennial motion of the earth's perihelion in Newcomb's theory of the sun be modified to include an improved value for precession and to replace an empirical term with a physical constant. Oort (Ref. 30) had derived a new value for the general precession in longitude referred to the FK3, and it differed by 1″83/century from that which Newcomb embodied in his theory. Moreover, to account for discrepancies between Newtonian theories of motion and observations of the inner planets, Newcomb incremented the secular motion of each perihelion by 8.06×10^{-8} times the centennial mean motion of the individual planet. This he explained as a consequence of a presumed small deviation from the $1/r^2$ law of gravitation. It is now known that the theory of general relativity predicts a 3″84/century perihelion advance; therefore, Clemence proposed changes of 1″83 for correction of the general precession, 3″84 for the relativistic effect, and −10″45 for the removal of Newcomb's empirical increment. These total −4″78.

To maintain the same mean longitude for the sun, so as not to affect the definition and determination of UT, he further suggested adding 4″78 to the centennial increase in the mean anomaly of the earth. Herget's evaluation of the Tables of the Sun (see Ref. 12) incorporates this correction.

P. M. Janiczek of the United States Naval Observatory has shown that there are discordances between the theory as published (see Ref. 32) and that previously developed. His comparisons and the discussion by Clemence (Ref. 31) indicate that Newcomb not only neglected a number of terms with small coefficients in constructing his tables, but also included terms in the tables that were not in the theory presented in Ref. 32.

The replacements in the theory in Table A-1 will increase agreement with Ref. 12 in longitude and radius vector.

Table A-1. Replacements in Newcomb's theory of the sun

g_{Venus}	g_{earth}	g_{Mars}	ν_c	ν_s	ρ_e	ρ_s
−2	0		0″000	0″000		
−3	2		−0″013	0″000		
−4	3		0″000	0″000		
−5	8		0″154	0″000		
−7	10		−0″002	−0″002		
−8	9		0″002	−0″003		
−8	12		−0″033	−0″054		
−8	14		0″000	0″000		
−10	10		0″000	0″000		
	−1	2	−1″659	−0″617		
	−4	4	0″011	0″032		
	−7	11	0″000	0″000	17	−10

Errata already published are the replacement of argument −2,2 by −3,2 (Ref. 32, p. 17) for the Venus perturbation in latitude, and the sign change to −(1″882−0″016T) in the long-period inequalities.

To facilitate evaluation of the thus-amended theory, since the program was generalized for evaluating other planetary theories, the long-period perturbations were added to the mean anomaly after computation of the equation of center.

The difference between the longitude in Ref. 12 and that derived from the theory with only the perihelion correction has roughly a 1-yr period and an amplitude of 0″4; therefore, the maximum expected discrepancy in the computed position of (48) Doris implementing only the perihelion term would be about 0″2. When all of the corrections are included, the agreement increases to about 0″10.

Appendix B

Eckert–Brouwer Differential Correction Coefficients

The Eckert–Brouwer differential correction coefficients in Tables B-1 through B-3 are the partial derivatives of the elliptic coordinates and velocities with respect to three sets of six functions ξ_i of the equatorial Keplerian elements. In terms of the semimajor axis a, the eccentricity e, the inclination I, the longitude of ascending node Ω, the argument of periapsis ω, and the mean anomaly M_0,

$$\Delta I = \Delta p \cos\omega - \Delta q \sin\omega \tag{B-1}$$

$$\sin I \, \Delta\Omega = \Delta p \sin\omega + \Delta q \cos\omega \tag{B-2}$$

$$\Delta\omega + \cos I \, \Delta\Omega = \Delta r \tag{B-3}$$

$$\Delta\psi_1 = \left| \mathbf{P}_x \Delta p + \mathbf{Q}_x \Delta q + \mathbf{R}_x \Delta r \right| \tag{B-4}$$

$$\Delta\psi_2 = \left| \mathbf{P}_y \Delta p + \mathbf{Q}_y \Delta q + \mathbf{R}_y \Delta r \right| \tag{B-5}$$

$$\Delta\psi_3 = \left| \mathbf{P}_z \Delta p + \mathbf{Q}_z \Delta q + \mathbf{R}_z \Delta r \right| \tag{B-6}$$

where **P**, **Q**, and **R** are the usual vectorial orbital constants.

In terms of the Keplerian radial distance r and velocity $\dot{r}$,

$$H = \frac{r - a(1 + e^2)}{ae(1 - e^2)} \tag{B-7}$$

$$K = \frac{r\dot{r}}{a^2 n^2 e}\left[1 + \frac{r}{a(1 - e^2)}\right] \tag{B-8}$$

$$H' = r\dot{r}\,\frac{r^2 - a[r + a(1 - e^2)]}{er^3 a(1 - e^2)} \tag{B-9}$$

$$K' = \frac{a - r}{ea(1 - e^2)} \tag{B-10}$$

Section V contains a discussion of the choice of values for all of the quantities used to generate and evaluate the expressions.

Set 1 is the basic set of coefficients. Set 2 is a modification designed to increase the separability of $\Delta\omega$ and M_0 for orbits with low eccentricity. Set 3 requires more calculation than do the others, but has the advantage of yielding a determinate solution regardless of the values of eccentricity or inclination.

When the normal equations are solved, the corrections to the elements may be obtained by premultiplying the matrix of parameters $\boldsymbol{\xi}$ by the matrix **G** given below:

$$\mathbf{G} = \begin{array}{c|cccccc} & \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 \\ \hline \Delta a & 0 & 0 & 0 & 0 & a & 0 \\ \Delta e & 0 & 0 & 0 & 0 & 0 & 1 \\ \Delta I & \multicolumn{4}{c}{} & 0 & 0 \\ \Delta\omega & \multicolumn{4}{c}{AB^{-1}_{(i)}} & 0 & 0 \\ \Delta\Omega & \multicolumn{4}{c}{} & 0 & 0 \\ \Delta M_0 & \multicolumn{4}{c}{} & 0 & 0 \end{array} \tag{B-11}$$

$$A = \begin{array}{c|cccc} & \Delta p & \Delta q & \Delta r & \Delta M_0 \\ \hline \Delta I & \cos\omega & -\sin\omega & 0 & 0 \\ \Delta\omega & -\dfrac{\cos I}{\sin I}\sin\omega & -\dfrac{\cos I}{\sin I}\cos\omega & 1 & 0 \\ \Delta\Omega & \dfrac{\sin\omega}{\sin I} & \dfrac{\cos\omega}{\sin I} & 0 & 0 \\ \Delta M_0 & 0 & 0 & 0 & 0 \end{array} \tag{B-12}$$

$$B_{(1)} = \begin{array}{c|cccc} & \Delta p & \Delta q & \Delta r & \Delta M_0 \\ \hline \Delta M_0 & 0 & 0 & 0 & 1 \\ \Delta\psi_1 & \mathbf{P}_x & \mathbf{Q}_x & \mathbf{R}_x & 0 \\ \Delta\psi_2 & \mathbf{P}_y & \mathbf{Q}_y & \mathbf{R}_y & 0 \\ \Delta\psi_3 & \mathbf{P}_z & \mathbf{Q}_z & \mathbf{R}_z & 0 \end{array} \tag{B-13}$$

$$B_{(2)} = \begin{array}{c|cccc} & \Delta p & \Delta q & \Delta r & \Delta M_0 \\ \hline \Delta M_0 + \Delta\psi_3 & \mathbf{P}_z & \mathbf{Q}_z & \mathbf{R}_z & 1 \\ \Delta\psi_1 & \mathbf{P}_x & \mathbf{Q}_x & \mathbf{R}_x & 0 \\ \Delta\psi_2 & \mathbf{P}_y & \mathbf{Q}_y & \mathbf{R}_y & 0 \\ e\Delta\psi_3 & e\mathbf{P}_z & e\mathbf{Q}_z & e\mathbf{R}_z & 0 \end{array} \tag{B-14}$$

$$
B_{(3)} = \begin{array}{r|cccc}
 & \Delta p & \Delta q & \Delta r & \Delta M_0 \\
\hline
\Delta M_0 + \Delta r & 0 & 0 & 1 & 1 \\
\Delta p & 1 & 0 & 0 & 0 \\
\Delta q & 0 & 1 & 0 & 0 \\
e\Delta r & 0 & 0 & e & 0
\end{array} \qquad \text{(B-15)}
$$

Use of matrix techniques also facilitates the determination of probable errors for the corrections to the elements, as the covariance of the corrections Γ_E is given in terms of the covariance Γ_ξ of the solution parameters by

$$
\Gamma_E = \mathbf{G}\Gamma_\xi\mathbf{G}^T \qquad \text{(B-16)}
$$

Table B-1. Eckert–Brouwer set 1

	$\Delta\psi_0$	$\Delta\psi_1$	$\Delta\psi_2$	$\Delta\psi_3$	$\frac{\Delta a}{a}$	Δe
Δx	$\frac{\dot{x}}{n}$	0	z	$-y$	$x - \frac{3}{2}t\dot{x}$	$Hx + K\dot{x}$
Δy	$\frac{\dot{y}}{n}$	$-z$	0	x	$y - \frac{3}{2}t\dot{y}$	$Hy + K\dot{y}$
Δz	$\frac{\dot{z}}{n}$	y	$-x$	0	$z - \frac{3}{2}t\dot{z}$	$Hz + K\dot{z}$
$\Delta\dot{x}$	$\frac{\ddot{x}}{n}$	0	$\dot{z}$	$-\dot{y}$	$-\frac{1}{2}(\dot{x} + 3\ddot{x})$	$H'x + K'\dot{x}$
$\Delta\dot{y}$	$\frac{\ddot{y}}{n}$	$-\dot{z}$	0	$\dot{x}$	$-\frac{1}{2}(\dot{y} + 3\ddot{y})$	$H'y + K'\dot{y}$
$\Delta\dot{z}$	$\frac{\ddot{z}}{n}$	$\dot{y}$	$-\dot{x}$	0	$-\frac{1}{2}(\dot{z} + 3\ddot{z})$	$H'z + K'\dot{z}$

Table B-2. Eckert–Brouwer set 2

	$\Delta M_0 + \Delta\psi_3$	$\Delta\psi_1$	$\Delta\psi_2$	$e\Delta\psi_3$	$\frac{\Delta a}{a}$	Δe
Δx	$\frac{\dot{x}}{n}$	0	z	$-\frac{1}{e}\left(\frac{\dot{x}}{n} + y\right)$	$x - \frac{3}{2}t\dot{x}$	$Hx + K\dot{x}$
Δy	$\frac{\dot{y}}{n}$	$-z$	0	$-\frac{1}{e}\left(\frac{\dot{y}}{n} - x\right)$	$y - \frac{3}{2}t\dot{y}$	$Hy + K\dot{y}$
Δz	$\frac{\dot{z}}{n}$	y	$-x$	$-\frac{1}{e}\frac{\dot{z}}{n}$	$z - \frac{3}{2}t\dot{z}$	$Hz + K\dot{z}$
$\Delta\dot{x}$	$\frac{\ddot{x}}{n}$	0	$\dot{z}$	$-\frac{1}{e}\left(\frac{\ddot{x}}{n} + \dot{y}\right)$	$-\frac{1}{2}(\dot{x} + 3\ddot{x})$	$K'x + K'\dot{x}$
$\Delta\dot{y}$	$\frac{\ddot{y}}{n}$	$-\dot{z}$	0	$-\frac{1}{e}\left(\frac{\ddot{y}}{n} - \dot{x}\right)$	$-\frac{1}{2}(\dot{y} + 3\ddot{y})$	$H'y + K'\dot{y}$
$\Delta\dot{z}$	$\frac{\ddot{z}}{n}$	$\dot{y}$	$-\dot{x}$	$-\frac{1}{e}\frac{\ddot{z}}{n}$	$-\frac{1}{2}(\dot{z} + 3\ddot{z})$	$H'z + K'\dot{z}$

Table B-3. Eckert–Brouwer set 3

	$\Delta M_0 + \Delta r$	Δp	Δq	$e\Delta r$	$\frac{\Delta a}{a}$	Δe
Δx	$\frac{\dot{x}}{n}$	$\mathbf{P}_y z - \mathbf{P}_z y$	$\mathbf{Q}_y z - \mathbf{Q}_z y$	$\frac{1}{e}\left(\mathbf{R}_y z - \mathbf{R}_z y - \frac{x}{n}\right)$	$x - \frac{3}{2} t\dot{x}$	$Hx + K\dot{x}$
Δy	$\frac{\dot{y}}{n}$	$\mathbf{P}_z x - \mathbf{P}_x z$	$\mathbf{Q}_z x - \mathbf{Q}_x z$	$\frac{1}{e}\left(\mathbf{R}_z x - \mathbf{R}_x z - \frac{y}{n}\right)$	$y - \frac{3}{2} t\dot{y}$	$Hy + K\dot{y}$
Δz	$\frac{\dot{z}}{n}$	$\mathbf{P}_x y - \mathbf{P}_y x$	$\mathbf{Q}_x y - \mathbf{Q}_y x$	$\frac{1}{e}\left(\mathbf{R}_x y - \mathbf{R}_y x - \frac{z}{n}\right)$	$z - \frac{3}{2} t\dot{z}$	$Hz + K\dot{z}$
$\Delta \dot{x}$	$\frac{\ddot{x}}{n}$	$\mathbf{P}_y \dot{z} - \mathbf{P}_z \dot{y}$	$\mathbf{Q}_y \dot{z} - \mathbf{Q}_z \dot{y}$	$\frac{1}{e}\left(\mathbf{R}_y \dot{z} - \mathbf{R}_z \dot{y} - \frac{\ddot{x}}{n}\right)$	$-\frac{1}{2}(\dot{x} + 3\ddot{x})$	$H'x + K'\dot{x}$
$\Delta \dot{y}$	$\frac{\ddot{y}}{n}$	$\mathbf{P}_z \dot{x} - \mathbf{P}_x \dot{z}$	$\mathbf{Q}_z \dot{x} - \mathbf{Q}_x \dot{z}$	$\frac{1}{e}\left(\mathbf{R}_z \dot{x} - \mathbf{R}_x \dot{z} - \frac{\ddot{y}}{n}\right)$	$-\frac{1}{2}(\dot{y} + 3\ddot{y})$	$H'y + K'\dot{y}$
$\Delta \dot{z}$	$\frac{\ddot{z}}{n}$	$\mathbf{P}_x \dot{y} - \mathbf{P}_y \dot{x}$	$\mathbf{Q}_x \dot{y} - \mathbf{Q}_y \dot{x}$	$\frac{1}{e}\left(\mathbf{R}_x \dot{y} - \mathbf{R}_y \dot{x} - \frac{\ddot{z}}{n}\right)$	$-\frac{1}{2}(\dot{z} + 3\ddot{z})$	$H'z + K'\dot{z}$

Appendix C

Observations and Residuals

This appendix lists the observations used in this report. The various columns in the printout contain the following information:

(1) International Astronomical Union observatory number. Negative numbers are used to identify observatories that have not been assigned an IAU number. (See Table 3 for the names and locations of the observatories.)

(2) Year, month, and day of the observations in ephemeris time.

(3) Reduced 1950.0 coordinates.

(4) Corrections (if any) to the FK4 system.

(5) Residuals from the reference orbit (before) and the linearized residuals after the solution, using $\partial \mathbf{r}/\partial m$ and set 3.

(6) Type of observation: P = photographic; V = visual; R = rereduced; M = meridian.

OBS	DATE	R.A.	DEC.	FK4-CAT. R.A.	FK4-CAT. DEC.	(O-C) R.A. BEFORE	(O-C) R.A. AFTER	(O-C) DEC. BEFORE	(O-C) DEC. AFTER	TYPE
		H M S	0 / //	S	//	//	//	//	//	
786	1967 10 30.11257	23 17 48.081	-04 18 15.25	0.033	0.37	1.562	1.562	1.09	1.09	P
786	1967 10 30.07438	23 17 48.440	-04 18 09.34	0.033	0.37	0.896	0.896	0.39	0.39	P
990	1967 10 06.84695	23 26 36.377	-02 34 34.92	0.019	0.04	4.874	4.874	7.71	7.71	P
990	1967 10 05.89903	23 27 08.885	-02 29 09.02	0.019	0.04	0.209	0.209	1.91	1.91	P
990	1967 10 05.87820	23 27 09.196	-02 29 07.02	0.019	0.04	-6.470	-6.470	-3.44	-3.44	P
786	1967 09 02.29243	23 49 20.495	01 00 17.62	-0.000	-0.00	0.218	0.218	-0.09	-0.09	P
786	1967 09 02.25980	23 49 21.698	01 00 29.13	0.003	0.31	0.009	0.009	0.56	0.56	P
786	1967 08 15.32785	23 57 46.756	02 20 41.63	0.003	0.31	0.129	0.128	1.22	1.22	P
786	1967 08 15.30563	23 57 47.256	02 20 44.43	0.003	0.31	0.812	0.812	-0.34	-0.34	P
95	1966 07 20.87677	18 24 55.903	-14 09 41.28	-0.000	-0.00	1.476	1.477	1.21	1.21	P
95	1966 07 19.88064	18 25 36.113	-14 07 46.98	-0.000	-0.00	1.753	1.755	5.13	5.13	P
95	1966 07 16.93348	18 27 39.003	-14 02 49.08	-0.000	-0.00	-1.639	-1.638	-8.95	-8.95	P
786	1966 07 14.16743	18 29 39.937	-13 58 06.94	0.004	0.04	0.907	0.908	1.44	1.44	P
786	1966 07 14.15216	18 29 40.606	-13 58 05.85	0.004	0.04	0.485	0.486	1.08	1.08	P
95	1966 07 12.89119	18 30 36.892	-13 56 12.78	-0.000	-0.00	-0.469	-0.468	-2.21	-2.21	P
95	1966 06 24.94449	18 44 39.412	-13 37 48.27	-0.000	-0.00	3.491	3.492	0.51	0.51	P
786	1966 06 23.22438	18 45 58.767	-13 37 02.88	0.004	0.04	0.808	0.809	0.66	0.66	P
786	1966 06 23.21327	18 45 59.256	-13 37 02.28	0.004	0.04	0.285	0.286	1.00	1.00	P
95	1966 06 20.93487	18 47 42.886	-13 36 20.33	0.004	0.04	2.793	2.794	1.43	1.43	P
786	1966 06 16.25980	18 51 06.707	-13 36 00.03	0.004	0.04	1.114	1.116	0.52	0.52	P
786	1966 06 16.23757	18 51 07.677	-13 36 00.02	0.004	0.04	1.184	1.186	0.63	0.63	P
95	1966 06 15.95867	18 51 19.233	-13 36 01.47	-0.000	-0.00	-0.939	-0.938	0.95	0.95	P
786	1965 05 27.19173	13 31 03.516	-04 38 16.14	-0.009	0.33	0.279	0.288	0.45	0.45	P
786	1965 05 27.17575	13 31 03.815	-04 38 18.04	-0.009	0.33	0.257	0.265	0.47	0.47	P
786	1965 05 27.13704	13 31 04.585	-04 38 22.24	-0.009	0.33	0.750	0.759	0.96	0.96	P
786	1965 05 27.12366	13 31 04.775	-04 38 23.53	-0.009	0.33	-0.221	-0.212	1.28	1.28	P
786	1965 05 18.14729	13 34 29.365	-05 01 49.50	-0.009	0.33	4.672	4.682	-0.12	-0.12	P
786	1965 05 18.14729	13 34 29.365	-05 01 49.32	-0.009	0.33	4.672	4.682	0.06	0.06	P
420	1965 05 03.55685	13 42 41.116	-06 00 45.88	-0.009	0.33	0.364	0.374	0.30	0.30	P
95	1965 05 02.96996	13 43 03.955	-06 03 37.41	-0.000	-0.00	-1.681	-1.671	3.79	3.79	P
786	1965 05 01.19937	13 44 14.907	-06 12 25.18	-0.009	0.33	0.393	0.402	0.25	0.25	P
786	1965 05 01.17854	13 44 15.757	-06 12 31.78	-0.009	0.33	0.116	0.125	-0.08	-0.08	P
420	1965 04 13.61796	13 56 42.367	-07 48 37.92	-0.000	-0.00	0.061	0.071	0.06	0.06	P
388	1964 02 12.51152	08 26 58.618	11 19 08.76	-0.000	-0.00	3.664	3.688	-0.32	-0.32	P
388	1964 02 12.47194	08 27 00.188	11 18 57.36	-0.000	-0.00	1.161	1.185	-0.45	-0.45	P
786	1964 02 09.17506	08 29 24.148	11 03 28.24	-0.050	0.17	-1.180	-1.156	0.20	0.20	P
6	1964 02 01.89198	08 35 03.239	10 30 03.76	-0.000	-0.00	-0.851	-0.827	-0.84	-0.84	P
330	1964 01 19.82153	08 45 25.709	09 37 28.86	-0.000	-0.00	-1.191	-1.167	2.34	2.34	P
786	1964 01 16.34590	08 48 02.394	09 25 55.09	-0.046	0.13	-1.198	-1.174	0.25	0.25	P
786	1964 01 16.30354	08 48 04.344	09 25 47.54	-0.046	0.13	-0.753	-0.729	0.59	0.59	P
13	1962 11 30.84322	02 07 37.509	06 39 36.04	-0.023	-0.05	-0.838	-0.823	-1.15	-1.15	P
13	1962 11 30.83906	02 07 37.649	06 39 36.63	-0.023	-0.05	-0.380	-0.364	-1.15	-1.15	P
13	1962 11 30.82867	02 07 37.919	06 39 38.03	-0.023	-0.05	-0.425	-0.409	-1.21	-1.21	P
13	1962 11 30.82244	02 07 38.099	06 39 38.82	-0.023	-0.05	-0.180	-0.164	-1.30	-1.30	P
330	1962 11 28.72274	02 08 33.552	06 44 58.19	-0.000	-0.00	0.336	0.352	0.27	0.27	P
334	1962 11 20.58446	02 12 55.791	07 11 23.39	-0.000	-0.00	0.670	0.686	-0.38	-0.38	P
388	1962 11 19.47400	02 13 36.811	07 15 36.79	-0.000	-0.00	2.235	2.252	-4.75	-4.75	P
330	1962 10 25.69676	02 31 32.380	09 21 10.68	-0.000	-0.00	2.818	2.835	0.25	0.25	P
13	1962 10 23.01237	02 33 30.240	09 36 28.69	-0.030	0.29	-0.229	-0.211	0.31	0.31	P
13	1962 10 23.00891	02 33 30.359	09 36 29.96	-0.030	0.29	-0.742	-0.725	0.39	0.39	P

OBS	DATE	R.A.	DEC.	FK4-CAT. R.A.	FK4-CAT. DEC.	(O-C) R.A. BEFORE	(O-C) R.A. AFTER	(O-C) DEC. BEFORE	(O-C) DEC. AFTER	TYPE
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13	1962 10 22.99471	02 33 31.049	09 36 34.78	-0.030	0.29	0.124	0.141	0.35	0.35	P
13	1962 10 22.99055	02 33 31.229	09 36 36.14	-0.030	0.29	0.045	0.063	0.29	0.29	P
786	1961 09 16.09206	20 41 19.838	-12 14 26.28	0.019	-0.01	0.821	0.814	0.88	0.88	P
786	1961 09 16.02956	20 41 20.859	-12 14 12.91	0.019	-0.01	0.937	0.929	0.44	0.44	P
760	1961 09 03.26192	20 46 10.205	-11 22 01.26	0.026	-0.15	1.773	1.764	0.96	0.96	P
760	1961 09 03.21945	20 46 11.465	-11 21 50.31	0.026	-0.15	0.981	0.973	0.68	0.68	P
388	1961 08 16.52886	20 57 09.972	-09 59 14.71	0.024	0.53	5.600	5.591	3.22	3.22	P
388	1961 08 16.51497	20 57 10.603	-09 59 10.61	0.024	0.53	5.879	5.871	3.37	3.37	P
786	1960 06 28.13441	15 52 24.253	-12 01 10.37	-0.014	-0.03	-0.942	-0.941	1.14	1.14	P
786	1960 06 28.09830	15 52 25.232	-12 01 11.51	-0.014	-0.03	-0.810	-0.809	0.59	0.59	P
8	1960 06 20.86500	15 55 58.943	-12 05 34.15	-0.014	-0.03	0.026	0.028	0.08	0.08	P
420	1960 06 09.52443	16 03 09.172	-12 21 43.35	-0.014	-0.03	-1.561	-1.559	-0.83	-0.83	P
420	1960 05 09.65400	16 26 13.806	-13 48 31.77	-0.013	-0.05	-0.451	-0.450	-0.16	-0.16	P
786	1959 04 07.12677	11 09 50.086	03 55 29.06	-0.044	0.04	-0.282	-0.211	0.31	0.31	P
786	1959 04 07.09969	11 09 50.886	03 55 20.63	-0.044	0.04	-0.414	-0.344	0.14	0.14	P
6	1959 04 02.83355	11 12 01.821	03 32 40.25	-0.000	-0.00	1.240	1.311	-0.85	-0.85	P
760	1959 04 01.18201	11 12 56.971	03 23 22.21	0.000	0.00	4.898	4.970	-4.13	-4.13	P
760	1959 04 01.13097	11 12 58.446	03 23 07.96	-0.044	0.04	-0.001	0.071	-0.98	-0.98	P
786	1959 03 17.18163	11 22 39.149	01 51 02.32	-0.042	0.13	-0.520	-0.446	-0.02	-0.02	P
786	1959 03 17.14760	11 22 40.649	01 50 49.39	-0.042	0.13	-0.239	-0.165	0.24	0.24	P
24	1959 03 09.01663	11 28 27.572	00 58 19.75	-0.030	0.12	-0.407	-0.333	-6.26	-6.26	P
786	1959 03 05.17191	11 31 09.833	00 34 19.57	-0.030	0.12	-0.875	-0.801	0.16	0.16	P
786	1959 03 05.14413	11 31 11.012	00 34 09.37	-0.030	0.12	-1.007	-0.934	0.25	0.25	P
330	1959 03 03.70581	11 32 10.603	00 25 20.23	-0.000	-0.00	2.116	2.190	-0.55	-0.55	P
786	1958 01 20.11635	05 14 29.727	13 57 13.17	-0.001	0.20	-0.129	-0.021	1.23	1.23	P
786	1958 01 20.06844	05 14 30.867	13 57 07.50	-0.001	0.20	0.129	0.237	0.79	0.79	P
786	1958 01 10.13371	05 19 18.257	13 42 29.67	-0.001	0.20	-0.031	0.082	1.21	1.21	P
786	1958 01 10.09969	05 19 19.487	13 42 27.46	-0.001	0.20	-0.384	-0.271	1.30	1.30	P
786	1957 12 16.18405	05 38 29.366	13 39 11.33	-0.001	0.20	-0.060	0.059	1.06	1.06	P
786	1957 12 16.16113	05 38 30.556	13 39 12.38	-0.001	0.20	-0.463	-0.344	0.96	0.96	P
990	1957 11 27.96148	05 53 04.216	14 06 27.41	-0.001	0.20	-1.889	-1.774	2.49	2.49	P
990	1957 11 27.94065	05 53 05.315	14 06 27.13	0.038	0.32	1.301	1.417	-0.30	-0.30	P
13	1956 10 15.87527	23 19 58.385	-03 30 06.70	0.038	0.32	0.732	0.746	-0.11	-0.11	P
13	1956 10 15.87109	23 19 58.536	-03 30 05.12	0.038	0.32	1.381	1.395	0.29	0.29	P
24	1956 10 10.91633	23 22 12.646	-03 05 08.83	0.038	0.32	-1.065	-1.051	2.12	2.12	P
786	1956 10 08.12606	23 23 39.116	-02 49 55.62	0.038	0.32	0.941	0.956	0.69	0.69	P
786	1956 10 08.09759	23 23 40.095	-02 49 45.63	0.038	0.32	1.306	1.321	1.09	1.09	P
13	1956 10 07.85613	23 23 47.936	-02 48 25.55	0.038	0.32	1.775	1.790	0.43	0.43	P
13	1956 10 07.85195	23 23 48.064	-02 48 24.11	0.038	0.32	1.621	1.635	0.45	0.45	P
24	1956 10 01.89829	23 27 15.234	-02 13 17.84	0.038	0.32	-0.386	-0.371	1.58	1.58	P
786	1956 10 01.15106	23 27 43.125	-02 08 42.35	0.038	0.32	0.931	0.945	0.74	0.74	P
786	1956 10 01.12884	23 27 43.995	-02 08 34.15	0.038	0.32	1.046	1.061	0.70	0.70	P
990	1956 09 28.93648	23 29 07.595	-01 54 53.81	0.019	0.05	2.107	2.122	-0.09	-0.09	P
990	1956 09 26.86704	23 30 28.465	-01 41 46.11	0.019	0.05	-3.294	-3.279	-1.31	-1.31	P
13	1956 09 24.85462	23 31 49.305	-01 28 48.64	0.019	0.05	-0.035	-0.020	-0.26	-0.26	P
13	1956 09 24.84767	23 31 49.575	-01 28 46.02	0.019	0.05	-0.296	-0.281	-0.34	-0.34	P
13	1956 09 17.01071	23 37 13.246	-00 37 40.17	0.001	0.34	-0.353	-0.339	-0.11	-0.11	P
13	1956 09 17.00446	23 37 13.507	-00 37 37.66	0.001	0.34	-0.470	-0.455	-0.04	-0.04	P
13	1956 09 14.99029	23 38 37.496	-00 24 33.20	0.001	0.34	-0.708	-0.693	-0.13	-0.13	P
13	1956 09 14.98334	23 38 37.817	-00 24 30.41	0.001	0.34	-0.355	-0.340	-0.04	-0.04	P

OBS	DATE	R.A.	DEC.	FK4-CAT.		(O-C)		(O-C)		TYPE
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24	1956 09 14.02241	23 39 17.767	-00 18 18.53	0.001	0.34	0.542	0.557	-0.30	-0.30	P
330	1956 09 10.76295	23 41 32.075	00 02 29.63	-0.000	-0.00	2.332	2.347	0.79	0.79	P
24	1956 09 09.01277	23 42 42.967	00 13 25.47	0.001	0.34	0.665	0.680	1.52	1.52	P
13	1956 09 04.03036	23 45 57.725	00 43 23.16	-0.000	-0.00	-0.870	-0.855	-0.24	-0.24	P
13	1956 09 04.02341	23 45 58.015	00 43 25.84	0.001	0.34	-0.561	-0.546	0.01	0.01	P
990	1956 09 03.92190	23 46 01.906	00 44 02.56	0.001	0.34	-2.006	-1.991	0.72	0.72	P
990	1956 08 28.89759	23 49 37.187	01 17 15.11	0.003	0.29	-2.975	-2.961	-0.80	-0.80	P
990	1956 08 27.89481	23 50 10.268	01 22 22.81	0.003	0.29	-4.703	-4.689	-2.26	-2.26	P
13	1956 08 22.00284	23 53 07.608	01 50 11.24	0.003	0.29	-0.493	-0.479	-0.55	-0.55	P
13	1956 08 21.99936	23 53 07.677	01 50 11.74	0.003	0.29	-0.913	-0.899	-0.95	-0.95	P
786	1955 07 22.14759	18 19 37.049	-14 15 02.67	0.006	0.20	1.572	1.549	0.48	0.48	P
786	1955 07 22.12259	18 19 37.959	-14 14 59.41	0.006	0.20	0.349	0.326	0.97	0.97	P
990	1955 07 13.90244	18 25 18.989	-14 01 15.68	0.006	0.20	-3.746	-3.769	3.20	3.20	P
990	1955 07 12.92884	18 26 02.399	-13 59 53.94	-0.044	0.04	-1.280	-1.303	-0.09	-0.09	P
990	1955 07 12.90801	18 26 03.176	-13 59 52.63	0.004	0.05	-3.951	-3.974	-0.60	-0.60	P
786	1955 07 12.17744	18 26 36.197	-13 58 49.16	0.004	0.05	0.663	0.640	0.92	0.92	P
786	1955 07 12.13579	18 26 38.156	-13 58 45.96	0.004	0.05	1.091	1.068	0.61	0.61	P
983	1955 07 11.97082	18 26 45.516	-13 58 33.13	0.004	0.05	0.452	0.429	-0.39	-0.39	P
760	1955 06 22.33191	18 42 07.267	-13 41 58.86	0.004	0.05	1.896	1.873	0.21	0.21	P
760	1955 06 22.29236	18 42 09.167	-13 42 00.58	0.004	0.05	2.411	2.388	-2.07	-2.07	P
786	1955 06 16.24550	18 46 37.786	-13 41 37.01	0.004	0.05	0.714	0.691	0.32	0.32	P
786	1955 06 16.22329	18 46 38.746	-13 41 36.82	0.004	0.05	0.453	0.430	0.68	0.68	P
786	1954 06 03.10140	13 28 54.405	-04 31 28.82	-0.009	0.33	-0.524	-0.495	-0.20	-0.20	P
786	1954 06 03.06390	13 28 54.835	-04 31 30.71	-0.009	0.33	-0.524	-0.494	0.43	0.43	P
786	1954 05 11.14376	13 37 35.566	-05 31 51.57	-0.009	0.33	-0.109	-0.075	0.50	0.50	P
786	1954 05 11.10487	13 37 36.936	-05 32 01.66	-0.009	0.33	-0.025	0.009	-0.13	-0.13	P
990	1954 05 05.89272	13 40 43.916	-05 54 38.78	-0.009	0.33	-0.688	-0.653	-0.66	-0.66	P
990	1954 05 04.88994	13 41 21.866	-05 59 14.28	-0.009	0.33	-4.197	-4.162	3.57	3.57	P
330	1954 04 27.54751	13 46 18.196	-06 35 58.21	-0.000	-0.00	-2.208	-2.172	2.03	2.03	P
12	1954 04 26.95314	13 46 43.001	-06 39 09.62	-0.024	0.19	-2.915	-2.879	-0.01	-0.01	P
330	1954 04 23.60376	13 49 05.986	-06 57 12.21	-0.000	-0.00	4.194	4.229	0.20	0.20	P
388	1954 04 23.53300	13 49 08.802	-06 57 35.72	-0.024	0.19	0.712	0.747	0.11	0.11	P
388	1954 03 29.62883	14 05 57.259	-09 15 32.73	-0.028	0.29	-0.747	-0.713	0.82	0.82	P
786	1954 03 29.26911	14 06 09.289	-09 17 23.52	-0.028	0.29	-1.022	-0.988	0.74	0.74	P
786	1954 03 29.23786	14 06 10.390	-09 17 32.51	-0.028	0.29	-0.786	-0.752	1.31	1.31	P
760	1953 03 07.18890	08 19 31.699	12 49 08.01	-0.050	0.17	0.514	0.625	0.57	0.57	P
760	1953 03 07.09655	08 19 33.288	12 48 45.10	-0.050	0.17	-0.676	-0.565	-0.67	-0.67	P
786	1953 02 19.15175	08 26 45.799	11 39 17.91	-0.050	0.17	-0.671	-0.552	0.77	0.77	P
786	1953 02 19.11980	08 26 46.988	11 39 08.56	-0.050	0.17	-1.017	-0.898	0.43	0.43	P
786	1953 02 14.18786	08 29 59.149	11 15 47.34	-0.050	0.17	-0.848	-0.727	0.78	0.78	P
786	1953 02 14.15592	08 30 00.499	11 15 38.04	-0.050	0.17	-1.026	-0.905	0.61	0.61	P
983	1953 02 07.98295	08 34 28.989	10 46 14.83	-0.050	0.17	-1.077	-0.954	-0.19	-0.19	P
786	1953 02 05.16668	08 36 38.919	10 33 03.57	-0.050	0.17	-0.640	-0.517	0.74	0.74	P
786	1953 02 05.14515	08 36 39.979	10 32 57.15	-0.050	0.17	-0.284	-0.161	0.34	0.34	P
73	1953 02 02.85306	08 38 28.099	10 22 24.73	-0.050	-0.03	1.490	1.613	1.51	1.51	P
786	1953 01 17.27571	08 51 29.785	09 15 46.11	-0.045	0.16	-0.828	-0.706	0.35	0.35	P
786	1953 01 17.23821	08 51 31.495	09 15 38.84	-0.045	0.16	-0.986	-0.864	0.51	0.51	P
388	1951 11 27.48367	02 09 27.242	06 52 19.79	-0.000	-0.00	0.641	0.697	0.51	0.51	P
22	1951 11 26.89704	02 09 44.651	06 54 03.99	-0.000	-0.00	2.282	2.338	1.61	1.61	P
22	1951 11 26.89704	02 09 44.812	06 54 03.59	-0.000	-0.00	4.683	4.739	1.21	1.21	P

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22	1951 11 25.87184	02 10 15.682	06 57 12.69	-0.000	-0.00	-1.845	-1.789	1.95	1.95	P
22	1951 11 25.87184	02 10 15.741	06 57 13.29	-0.000	-0.00	-0.946	-0.890	2.55	2.55	P
22	1951 11 23.87364	02 11 20.171	07 03 44.99	-0.000	-0.00	2.448	2.505	1.36	1.36	P
22	1951 11 23.87364	02 11 20.242	07 03 44.39	-0.000	-0.00	3.500	3.557	0.76	0.76	P
983	1951 11 19.84004	02 13 41.775	07 18 36.07	-0.056	-0.02	-1.277	-1.220	-0.25	-0.25	P
786	1951 11 19.13715	02 14 07.929	07 21 25.72	-0.022	0.11	-0.355	-0.298	1.08	1.08	P
786	1951 11 19.12395	02 14 08.449	07 21 28.95	-0.022	0.11	-0.289	-0.232	1.11	1.11	P
388	1951 11 08.63992	02 21 21.651	08 09 52.48	-0.000	-0.00	1.520	1.579	0.61	0.61	P
388	1951 11 08.59548	02 21 23.610	08 10 05.08	-0.000	-0.00	0.442	0.501	-0.48	-0.48	P
786	1951 11 04.18576	02 24 41.658	08 33 25.24	-0.022	0.11	-0.325	-0.266	0.93	0.93	P
786	1951 11 04.17326	02 24 42.268	08 33 29.08	-0.022	0.11	0.027	0.087	0.70	0.70	P
57	1951 10 31.87055	02 27 12.844	08 51 43.06	-0.056	-0.02	-0.753	-0.693	0.64	0.64	P
57	1951 10 31.87055	02 27 12.900	08 51 43.08	-0.000	-0.00	0.085	0.144	0.66	0.66	P
786	1951 10 26.20069	02 31 29.466	09 23 58.33	-0.024	0.29	0.116	0.175	1.01	1.01	P
786	1951 10 26.18680	02 31 30.105	09 24 02.67	-0.024	0.29	0.123	0.182	0.58	0.58	P
786	1951 10 10.27985	02 42 03.376	10 53 04.57	-0.022	0.09	0.611	0.668	0.41	0.41	P
786	1951 10 10.26249	02 42 03.951	10 53 10.00	-0.028	0.09	0.183	0.240	0.36	0.36	P
786	1950 09 15.10659	20 37 01.596	-12 20 53.66	0.016	-0.01	0.505	0.468	1.00	1.00	P
786	1950 09 15.08576	20 37 01.946	-12 20 49.09	0.016	-0.01	0.594	0.557	0.98	0.98	P
983	1950 08 08.93874	20 57 50.882	-09 37 10.52	0.023	0.52	1.160	1.120	1.44	1.44	P
388	1950 07 25.67569	21 08 13.627	-08 40 25.14	-0.000	-0.00	0.995	0.956	-2.74	-2.74	P
786	1949 05 24.23402	16 14 45.658	-13 04 33.09	-0.009	0.03	-1.218	-1.200	0.26	0.26	P
786	1949 05 24.21873	16 14 46.460	-13 04 35.29	-0.009	0.03	-0.167	-0.149	0.77	0.77	P
6	1949 05 20.91200	16 17 19.837	-13 14 37.52	-0.000	-0.00	-0.932	-0.915	-1.45	-1.45	P
388	1949 05 20.57603	16 17 35.568	-13 15 39.32	-0.000	-0.00	3.776	3.794	-0.98	-0.98	P
786	1948 04 23.16357	11 06 41.690	04 51 41.15	-0.000	-0.00	-0.382	-0.265	-0.53	-0.53	P
786	1948 04 23.12915	11 06 42.050	04 51 36.75	-0.000	-0.00	-0.315	-0.198	0.66	0.66	P
786	1948 04 16.19755	11 08 21.120	04 27 39.85	-0.000	-0.00	-0.914	-0.793	0.14	0.14	P
786	1948 04 16.15519	11 08 21.961	04 27 30.05	-0.000	-0.00	-0.736	-0.614	0.30	0.30	P
990	1948 03 09.96700	11 29 47.842	00 56 08.13	-0.000	-0.00	2.281	2.415	-0.77	-0.77	P
990	1948 03 09.94616	11 29 48.642	00 56 01.73	-0.000	-0.00	0.532	0.666	0.85	0.85	P
990	1948 03 08.92741	11 30 32.132	00 49 28.73	-0.000	-0.00	-0.304	-0.170	-1.15	-1.15	P
990	1948 03 08.90658	11 30 32.802	00 49 20.93	-0.000	-0.00	-3.902	-3.768	-0.97	-0.97	P
990	1948 03 06.93714	11 31 56.603	00 36 52.93	-0.000	-0.00	-1.347	-1.213	0.16	0.16	P
990	1948 03 06.91630	11 31 57.482	00 36 46.13	-0.000	-0.00	-1.684	-1.550	1.24	1.24	P
62	1948 03 03.01513	11 34 40.033	00 12 33.93	-0.000	-0.00	0.647	0.780	-1.53	-1.53	P
12	1948 02 18.19524	11 43 07.955	-01 02 31.34	-0.038	0.04	-0.028	0.100	0.34	0.34	P
754	1947 01 17.07190	05 13 01.604	13 49 37.59	-0.054	0.19	0.664	0.771	0.68	0.68	P
786	1946 12 14.21628	05 37 16.397	13 39 45.70	-0.000	-0.00	0.427	0.544	0.90	0.90	P
786	1946 12 14.20066	05 37 17.208	13 39 47.70	-0.000	-0.00	0.152	0.269	1.91	1.91	P
12	1945 10 04.88845	23 16 59.673	-03 06 48.61	-0.000	-0.00	0.204	0.136	0.75	0.75	P
62	1945 09 12.89351	23 31 25.307	-00 47 49.47	-0.000	-0.00	-2.755	-2.824	-0.07	-0.07	P
62	1945 09 11.90300	23 32 06.486	-00 41 29.47	-0.000	-0.00	-3.556	-3.625	-0.63	-0.63	P
28	1944 07 17.95065	18 16 36.793	-14 14 40.76	-0.000	-0.00	4.685	4.622	2.45	2.45	P
28	1943 04 05.96523	14 01 07.224	-08 38 19.14	-0.000	-0.00	3.978	4.028	-1.76	-1.76	P
28	1943 04 05.01593	14 01 43.914	-08 43 29.35	-0.000	-0.00	1.097	1.147	0.99	0.99	P
12	1943 04 04.94346	14 01 46.677	-08 43 54.52	-0.000	-0.00	-0.688	-0.638	-0.41	-0.41	P
804	1942 02 12.06622	08 29 07.028	11 10 31.88	-0.000	-0.00	2.056	2.173	1.60	1.60	V
804	1942 02 11.07750	08 29 49.578	11 05 48.93	-0.000	-0.00	2.210	2.327	0.75	0.75	V
804	1942 02 10.09854	08 30 32.396	11 01 09.37	-0.000	-0.00	2.708	2.825	0.10	0.10	V

OBS	DATE	R.A. H M S	DEC. ° ' "	FK4-CAT. R.A. S	FK4-CAT. DEC. "	(O-C) R.A. BEFORE "	(O-C) R.A. AFTER "	(O-C) DEC. BEFORE "	(O-C) DEC. AFTER "	TYPE
804	1942 02 07.06328	08 32 48.988	10 46 51.31	-0.000	-0.00	2.852	2.971	2.00	2.00	V
804	1942 02 05.07196	08 34 21.305	10 37 35.89	-0.000	-0.00	5.045	5.164	3.98	3.98	V
8	1942 02 04.89543	08 34 29.212	10 36 39.69	-0.077	0.33	0.464	0.583	1.75	1.75	P
804	1942 02 04.14933	08 35 03.962	10 33 20.86	-0.000	-0.00	-2.565	-2.446	4.59	4.59	V
804	1940 11 05.05306	02 11 57.895	07 46 07.87	-0.000	-0.00	4.480	4.419	-0.27	-0.27	V
804	1940 11 01.07198	02 14 55.491	08 08 03.01	-0.000	-0.00	2.676	2.615	0.92	0.92	V
804	1940 10 31.09106	02 15 39.732	08 13 33.58	-0.000	-0.00	2.109	2.048	-1.29	-1.29	V
804	1940 10 30.08268	02 16 25.364	08 19 21.84	-0.000	-0.00	1.798	1.736	1.88	1.88	V
874	1940 10 29.09358	02 17 10.232	08 25 01.80	-0.000	-0.00	2.891	2.829	0.66	0.66	V
62	1940 10 03.98428	02 33 48.719	10 47 48.97	-0.000	-0.00	-3.127	-3.187	-0.14	-0.14	P
62	1940 10 03.03598	02 34 16.989	10 52 35.87	-0.000	-0.00	7.259	7.200	-1.66	-1.66	P
78	1939 08 18.88491	20 42 23.180	-10 46 21.24	-0.000	-0.00	-0.387	-0.510	1.65	1.65	P
28	1939 08 17.90513	20 43 03.318	-10 41 57.53	0.033	0.37	-3.590	-3.714	3.38	3.38	P
28	1939 08 15.84055	20 44 29.706	-10 32 43.60	-0.006	0.25	-8.620	-8.744	-2.84	-2.84	P
28	1939 08 14.84588	20 45 12.245	-10 28 11.59	-0.006	0.05	-7.770	-7.894	0.44	0.44	P
12	1939 07 21.02395	21 03 15.124	-08 51 02.80	0.025	0.44	1.671	1.549	0.91	0.91	P
8	1938 06 16.95779	15 53 38.697	-12 04 38.27	-0.020	-0.03	-0.072	-0.088	-0.59	-0.59	P
8	1938 06 16.95225	15 53 38.907	-12 04 39.43	-0.020	-0.03	-0.024	-0.040	-1.30	-1.30	P
45	1938 06 08.91255	15 58 53.715	-12 18 03.35	-0.011	0.03	-4.771	-4.788	0.66	0.66	R
45	1938 06 05.98055	16 00 59.350	-12 24 13.81	-0.011	0.03	-2.979	-2.996	1.55	1.55	R
78	1938 06 01.84063	16 04 04.017	-12 33 59.62	-0.000	-0.00	-1.944	-1.961	0.09	0.09	P
45	1938 05 27.95841	16 07 48.601	-12 47 03.17	-0.008	0.02	-1.689	-1.706	1.43	1.43	R
45	1938 05 15.94683	16 17 02.158	-13 24 06.31	-0.008	0.02	-0.938	-0.955	1.64	1.64	R
45	1938 05 12.99621	16 19 11.914	-13 33 56.94	-0.008	0.02	0.462	0.445	1.79	1.79	R
45	1938 05 06.02919	16 23 57.888	-13 57 46.34	-0.000	-0.00	-4.823	-4.839	1.06	1.06	R
28	1938 05 06.00718	16 23 59.272	-13 57 56.28	-0.000	-0.00	2.572	2.555	-4.29	-4.29	P
73	1937 03 30.80367	11 09 29.775	03 33 16.49	-0.000	-0.00	-0.716	-0.647	0.62	0.62	P
73	1937 03 22.84780	11 14 22.711	02 45 31.61	-0.000	-0.00	2.202	2.273	0.41	0.41	P
73	1937 03 18.84278	11 17 04.797	02 20 04.41	-0.000	-0.00	2.656	2.727	-0.54	-0.54	P
73	1937 03 08.90957	11 24 07.295	01 15 43.52	-0.000	-0.00	0.618	0.690	1.06	1.06	P
990	1935 11 28.98916	05 34 04.517	13 58 34.30	-0.000	-0.00	1.040	0.984	-1.08	-1.08	P
990	1935 11 28.96138	05 34 05.588	13 58 37.80	-0.000	-0.00	-2.334	-2.390	-1.32	-1.32	P
990	1935 11 28.91971	05 34 07.587	13 58 42.90	-0.000	-0.00	-1.339	-1.395	-1.79	-1.79	P
338	1934 09 28.54449	23 08 18.920	-03 22 25.07	-0.000	-0.00	-1.380	-1.584	-0.61	-0.61	P
73	1934 09 27.87208	23 08 43.538	-03 18 21.05	-0.000	-0.00	-0.530	-0.735	-0.22	-0.22	P
73	1934 09 27.86827	23 08 43.669	-03 18 20.25	-0.000	-0.00	-0.755	-0.960	-0.81	-0.81	P
73	1934 09 27.86446	23 08 43.819	-03 18 18.45	-0.000	-0.00	-0.684	-0.889	-0.40	-0.40	P
35	1934 09 16.85576	23 16 00.323	-02 08 50.41	0.038	0.31	-1.806	-2.014	0.34	0.34	R
35	1934 09 16.85015	23 16 00.612	-02 08 48.81	0.038	0.31	-1.041	-1.249	-0.23	-0.23	R
990	1934 09 12.96902	23 18 42.638	-01 43 50.86	-0.000	-0.00	-5.578	-5.786	0.90	0.90	P
990	1934 09 12.94819	23 18 43.787	-01 43 42.66	-0.000	-0.00	-1.850	-2.058	1.09	1.09	P
990	1934 09 10.96902	23 20 06.947	-01 31 06.96	-0.000	-0.00	0.453	0.246	-0.29	-0.29	P
990	1934 09 10.94819	23 20 07.797	-01 30 58.46	-0.000	-0.00	-0.286	-0.494	0.27	0.27	P
73	1934 08 28.88715	23 28 50.447	-00 12 56.57	-0.000	-0.00	-0.075	-0.279	0.71	0.71	P
73	1934 08 28.88231	23 28 50.616	-00 12 55.37	-0.000	-0.00	-0.289	-0.493	0.32	0.32	P
990	1933 07 13.94680	18 12 45.384	-14 12 03.82	0.011	0.06	-0.786	-0.913	0.12	0.12	P
990	1933 07 13.92596	18 12 46.413	-14 12 02.32	0.011	0.06	1.047	0.921	-0.08	-0.08	P
338	1933 07 13.57749	18 13 01.199	-14 11 33.53	0.002	0.25	0.927	0.801	0.24	0.24	P
990	1933 07 11.95096	18 14 11.134	-14 09 27.22	0.011	0.06	0.102	-0.025	0.48	0.48	P
990	1933 07 11.93013	18 14 12.073	-14 09 27.12	0.011	0.06	0.215	0.088	-1.00	-1.00	P

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		H M S	0 / //	S	//	//	//	//	//	
990	1933 07 10.94125	18 14 55.604	-14 08 11.12	0.011	0.06	3.155	3.028	1.89	1.89	P
990	1933 07 10.92041	18 14 56.354	-14 08 10.92	0.011	0.06	0.237	0.110	0.56	0.56	P
78	1933 06 24.84789	18 27 27.822	-13 55 29.38	-0.000	-0.00	0.574	0.445	-0.31	-0.31	P
8	1933 06 24.01567	18 28 07.120	-13 55 17.60	0.002	0.19	-0.743	-0.871	-0.32	-0.32	P
78	1933 06 23.90049	18 28 12.832	-13 55 11.98	-0.000	-0.00	2.100	1.972	-0.39	-0.39	P
16	1932 04 27.94789	13 38 55.081	-06 07 36.97	-0.026	0.17	-0.536	-0.545	0.12	0.12	R
8	1931 01 28.93779	08 25 27.233	10 42 49.63	-0.050	-0.03	2.210	2.166	0.66	0.66	P
338	1931 01 19.58189	08 33 01.036	10 06 28.73	-0.034	-0.11	2.185	2.142	1.07	1.07	P
4	1929 11 27.92793	01 42 21.901	05 15 36.61	-0.057	-0.04	-0.513	-0.747	0.22	0.22	P
24	1929 11 21.78129	01 44 48.826	05 32 36.32	-0.018	-0.08	-1.989	-2.231	0.08	0.08	P
35	1929 11 06.78410	01 53 32.846	06 36 22.21	-0.012	-0.07	-1.806	-2.060	0.66	0.66	R
35	1929 11 06.77270	01 53 33.556	06 36 25.10	-0.012	-0.07	1.706	1.451	0.04	0.04	R
84	1929 10 07.04455	02 15 18.031	09 35 33.23	-0.020	0.10	-1.651	-1.903	0.32	0.32	R
78	1928 08 19.97135	20 31 30.565	-11 22 38.67	-0.000	-0.00	-1.205	-1.424	0.66	0.66	P
78	1928 08 19.92425	20 31 32.423	-11 22 26.61	-0.000	-0.00	-1.376	-1.594	0.49	0.49	P
20	1927 05 23.91798	16 02 40.830	-12 44 05.87	-0.006	0.02	2.022	1.922	0.57	0.57	R
8	1926 03 15.93140	11 06 02.730	02 56 46.81	-0.041	0.15	0.793	0.720	-1.36	-1.36	P
84	1926 03 06.86893	11 12 30.221	01 58 13.18	-0.000	-0.00	3.086	3.013	4.94	4.94	V
20	1926 03 04.89007	11 13 55.310	01 45 28.51	-0.000	-0.00	0.231	0.158	-2.89	-2.89	R
592	1926 02 22.99583	11 20 56.801	00 45 42.34	-0.030	0.14	-0.000	-0.000	-0.00	-0.00	Q
592	1926 02 22.99546	11 20 44.959	00 44 05.91	-0.030	0.14	-0.000	-0.000	-0.00	-0.00	Q
136	1923 10 14.72185	22 50 10.939	-05 29 32.88	0.035	0.38	0.446	0.154	-2.26	-2.26	R
14	1923 10 11.89484	22 51 08.767	-05 17 14.58	0.035	0.38	4.674	4.378	-2.32	-2.32	R
16	1923 09 13.88858	23 07 06.171	-02 36 45.06	0.004	0.32	0.953	0.635	-0.17	-0.17	R
84	1923 09 13.83926	23 07 08.000	-02 36 26.84	0.004	0.32	-1.957	-2.275	-0.21	-0.21	R
84	1923 09 11.96367	23 08 27.050	-02 24 33.90	0.004	0.32	6.182	5.863	0.01	0.01	R
136	1923 09 06.01055	23 12 36.207	-01 47 23.28	0.020	0.11	1.694	1.376	2.48	2.48	R
136	1923 09 05.97275	23 12 37.457	-01 47 10.57	0.020	0.11	-3.539	-3.857	1.33	1.33	R
136	1923 09 02.79535	23 14 48.339	-01 28 07.03	0.020	0.11	-2.670	-2.987	-4.49	-4.49	R
84	1923 08 30.90176	23 16 44.227	-01 11 12.96	-0.000	-0.00	-2.264	-2.579	-1.94	-1.94	R
84	1923 08 26.02679	23 19 50.190	-00 44 23.37	0.001	0.34	0.070	-0.242	-1.81	-1.81	R
20	1923 08 23.85599	23 21 08.521	-00 33 08.99	0.001	0.34	0.282	-0.027	-0.14	-0.14	R
20	1923 08 22.83982	23 21 43.982	-00 28 04.73	0.001	0.34	0.832	0.524	0.02	0.01	R
24	1923 08 15.00789	23 25 48.716	00 06 45.09	0.001	0.34	1.829	1.529	-5.09	-5.09	P
16	1922 08 16.89268	17 51 52.441	-15 19 03.97	0.003	0.08	0.416	0.247	-1.85	-1.85	R
16	1922 07 28.91149	17 56 52.832	-14 38 34.16	0.003	0.08	0.611	0.428	-2.13	-2.13	R
8	1922 07 26.90611	17 57 49.722	-14 34 50.09	-0.005	0.27	0.328	0.143	-0.18	-0.18	P
16	1922 07 26.90564	17 57 49.879	-14 34 50.11	0.003	0.08	2.261	2.076	-3.05	-3.05	R
16	1922 07 21.89451	18 00 30.627	-14 26 14.37	0.003	0.08	-0.752	-0.940	0.46	0.46	R
16	1922 07 20.89146	18 01 05.927	-14 24 39.47	0.003	0.08	0.203	0.015	-0.69	-0.69	R
20	1922 06 30.89931	18 15 19.891	-14 02 29.90	0.003	0.08	0.698	0.502	0.13	0.13	R
24	1922 06 29.97775	18 16 03.692	-14 01 59.47	0.011	0.06	2.903	2.706	-0.62	-0.62	P
20	1922 06 29.87001	18 16 08.849	-14 01 57.74	0.002	0.06	1.540	1.344	-2.70	-2.70	R
16	1922 06 21.92498	18 22 27.731	-13 59 25.92	0.002	0.06	2.788	2.591	1.42	1.42	R
14	1921 06 02.86443	13 14 25.174	-03 36 46.51	-0.006	0.38	-0.410	-0.508	-2.07	-2.07	R
84	1921 04 25.94919	13 30 15.004	-05 39 10.16	-0.009	0.32	-3.975	-4.090	2.35	2.35	R
84	1921 04 23.84659	13 31 41.528	-05 50 32.63	-0.009	0.32	-1.700	-1.815	0.72	0.72	R
24	1921 04 03.96193	13 45 42.615	-07 45 29.00	-0.021	0.22	-1.696	-1.810	-0.77	-0.77	P
24	1921 04 02.99379	13 46 21.225	-07 51 01.50	-0.021	0.22	-2.245	-2.359	0.51	0.51	P
24	1921 04 02.93718	13 46 23.635	-07 51 21.90	-0.021	0.22	-0.651	-0.765	-0.48	-0.48	P

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24	1920 02 18.86260	07 53 46.807	12 54 53.23	-0.054	0.17	1.415	1.192	-1.36	-1.36	P
14	1920 02 10.88018	07 58 23.073	12 21 00.44	-0.052	0.22	-3.883	-4.114	1.17	1.17	R
14	1920 02 08.82976	07 59 45.170	12 12 12.11	-0.052	0.22	-3.568	-3.801	0.81	0.81	R
14	1920 02 07.87860	08 00 24.705	12 08 06.34	-0.052	0.22	-0.054	-0.287	-0.57	-0.57	R
30	1918 12 21.76854	01 30 37.041	04 30 32.97	-0.015	0.04	-0.086	-0.382	0.74	0.74	R
30	1918 12 21.76854	01 30 37.263	04 30 26.52	-0.015	0.04	3.241	2.945	-5.70	-5.70	R
30	1918 12 20.74890	01 30 22.832	04 29 02.22	-0.015	0.04	-0.921	-1.219	2.08	2.08	R
30	1918 12 20.74890	01 30 23.004	04 28 59.69	-0.015	0.04	1.659	1.361	-0.45	-0.45	R
30	1918 11 22.83217	01 32 46.089	04 48 01.13	-0.015	0.04	0.919	0.570	-0.16	-0.16	R
30	1918 11 13.83507	01 37 01.085	05 19 50.92	-0.000	-0.00	3.189	2.826	-3.27	-3.27	V
30	1918 11 12.92236	01 37 31.633	05 23 45.81	-0.000	-0.00	4.024	3.659	-0.66	-0.66	V
24	1918 10 31.87822	01 45 18.265	06 24 01.11	-0.019	-0.08	-2.022	-2.400	0.04	0.04	P
30	1918 10 30.84654	01 46 02.607	06 29 47.54	-0.012	-0.06	1.101	0.723	-2.71	-2.71	R
30	1918 10 29.82555	01 46 46.556	06 35 47.77	-0.012	-0.06	-0.584	-0.963	7.69	7.69	R
30	1918 10 29.82555	01 46 46.520	06 35 37.87	-0.012	-0.06	-1.127	-1.505	-2.21	-2.21	R
30	1918 10 28.77306	01 47 32.276	06 41 43.96	-0.012	-0.06	-1.783	-2.163	-1.09	-1.09	R
30	1918 10 28.77306	01 47 32.277	06 41 44.87	-0.012	-0.06	-1.763	-2.142	-0.18	-0.18	R
30	1917 09 22.79261	20 13 57.004	-13 43 31.09	0.018	0.00	3.659	3.422	-0.41	-0.41	R
30	1917 09 21.78732	20 13 56.384	-13 40 48.53	0.018	0.00	3.753	3.515	1.02	1.02	R
30	1917 09 18.78167	20 14 02.080	-13 32 23.04	0.018	0.00	-0.250	-0.491	-1.27	-1.27	R
30	1917 09 17.79758	20 14 06.670	-13 29 26.22	0.018	0.00	1.245	1.002	1.03	1.03	R
30	1917 09 17.79758	20 14 06.617	-13 29 27.98	0.018	0.00	0.434	0.192	-0.74	-0.74	R
30	1917 09 16.80515	20 14 12.465	-13 26 27.41	0.018	0.00	0.985	0.741	-0.21	-0.21	R
30	1917 09 16.80515	20 14 12.530	-13 26 27.27	0.018	0.00	1.966	1.723	-0.06	-0.06	R
30	1917 08 22.83163	20 23 12.947	-11 53 19.63	0.025	-0.14	1.445	1.175	-1.08	-1.08	R
30	1917 08 22.83163	20 23 12.922	-11 53 18.67	0.025	-0.14	1.076	0.806	-0.12	-0.12	R
30	1917 08 21.88691	20 23 46.411	-11 49 21.68	0.025	-0.14	1.128	0.857	-1.71	-1.71	R
30	1917 08 21.88691	20 23 46.346	-11 49 20.42	0.025	-0.14	0.156	-0.114	-0.45	-0.45	R
-2	1917 08 21.82301	20 23 48.831	-11 49 03.47	0.025	-0.14	1.915	1.645	0.19	0.19	R
-2	1917 08 16.83934	20 26 57.904	-11 27 52.75	0.025	-0.14	0.584	0.310	1.81	1.81	R
8	1917 08 06.97039	20 33 56.521	-10 46 27.86	0.025	-0.14	-1.195	-1.474	-0.34	-0.34	R
24	1917 07 27.95947	20 41 26.986	-10 07 42.90	0.027	-0.16	1.744	1.466	1.51	1.51	P
8	1917 07 26.97829	20 42 10.947	-10 04 13.16	0.027	-0.16	2.237	1.959	0.83	0.83	P
8	1917 07 26.97366	20 42 11.307	-10 04 12.00	0.027	-0.16	2.373	2.095	0.37	0.37	P
8	1915 06 02.89352	10 53 33.107	06 33 18.14	-0.043	0.09	-0.877	-1.010	-0.28	-0.28	R
14	1915 04 13.89863	10 39 50.524	06 20 23.63	-0.000	-0.00	3.281	3.101	-5.56	-5.56	V
94	1915 03 18.87859	10 51 36.884	04 08 46.23	-0.000	-0.00	3.700	3.498	-1.16	-1.16	P
14	1915 03 13.92562	10 54 57.025	03 37 30.81	-0.000	-0.00	3.428	3.225	0.70	0.70	V
14	1915 03 12.95147	10 55 37.756	03 31 15.36	-0.000	-0.00	4.018	3.815	-0.09	-0.09	V
24	1915 03 09.94655	10 57 44.825	03 11 57.13	-0.045	0.09	-3.917	-4.120	2.59	2.59	P
24	1915 03 09.94648	10 57 44.785	03 11 56.53	-0.045	0.09	-4.566	-4.769	2.02	2.02	P
30	1914 01 04.82522	04 34 09.799	12 57 10.30	-0.046	0.12	1.090	0.731	0.18	0.18	R
30	1914 01 04.82522	04 34 09.680	12 57 10.36	-0.046	0.12	-0.692	-1.051	0.24	0.24	R
30	1914 01 03.80760	04 34 39.678	12 56 38.55	-0.046	0.12	1.981	1.621	0.22	0.22	R
30	1914 01 03.80760	04 34 39.560	12 56 40.41	-0.046	0.12	0.205	-0.155	2.07	2.07	R
30	1914 01 02.82216	04 35 09.839	12 56 13.69	-0.046	0.12	2.195	1.833	0.33	0.33	R
30	1914 01 02.82216	04 35 09.691	12 56 13.84	-0.046	0.12	-0.039	-0.401	0.49	0.49	R
30	1914 01 02.80510	04 35 10.359	12 56 13.36	-0.046	0.12	1.676	1.314	0.42	0.42	R
30	1913 12 26.82564	04 39 18.626	12 56 00.96	-0.046	0.12	0.912	0.539	-0.08	-0.08	R
30	1913 12 26.82564	04 39 18.483	12 56 00.80	-0.000	-0.00	-1.231	-1.604	-0.24	-0.24	R

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30	1913 12 25.82968	04 39 58.616	12 56 23.17	-0.046	0.12	1.286	0.911	0.20	0.20	R
30	1913 12 25.82968	04 39 58.651	12 56 22.82	-0.046	0.12	1.807	1.433	-0.15	-0.15	R
30	1913 12 22.83860	04 42 04.668	12 58 02.87	-0.046	0.12	2.906	2.528	-1.09	-1.09	R
30	1913 12 22.83860	04 42 04.593	12 58 02.42	-0.046	0.12	1.776	1.397	-1.53	-1.53	R
30	1913 12 21.82853	04 42 48.999	12 58 51.18	-0.046	0.12	1.942	1.562	1.37	1.37	R
30	1913 12 21.82853	04 42 48.894	12 58 50.04	-0.046	0.12	0.356	-0.024	0.23	0.23	R
30	1913 12 20.92565	04 43 29.083	12 59 36.93	-0.046	0.12	-0.797	-1.178	1.04	1.04	R
24	1913 12 19.94949	04 44 13.536	13 00 30.33	-0.046	0.12	-0.088	-0.470	-2.76	-2.76	R
30	1913 12 19.93911	04 44 14.016	13 00 32.81	-0.046	0.12	0.372	-0.010	1.40	1.40	R
30	1913 12 19.85237	04 44 18.098	13 00 36.81	-0.046	0.12	0.115	-0.267	0.32	0.32	R
45	1913 11 21.91786	05 07 08.481	13 59 14.54	-0.037	0.35	-1.365	-1.747	0.29	0.29	P
30	1912 09 21.82771	22 55 30.926	-03 55 55.84	0.037	0.30	0.471	0.120	-0.64	-0.64	R
30	1912 09 21.82771	22 55 30.949	-03 55 55.48	0.037	0.30	2.482	2.130	-3.28	-3.28	R
30	1912 09 18.84233	22 57 26.673	-03 37 37.19	0.037	0.30	-0.487	-0.840	-0.48	-0.48	R
30	1912 09 17.91011	22 58 03.632	-03 31 50.47	0.037	0.30	-1.128	-1.481	-0.76	-0.76	R
24	1911 05 26.98777	18 34 31.967	-14 21 53.61	0.015	0.07	-2.772	-2.996	-8.61	-8.61	P
45	1909 05 10.84170	08 18 26.082	15 17 00.47	-0.054	0.32	-0.593	-0.783	-1.09	-1.09	R
24	1909 04 21.87376	07 59 35.408	15 34 15.06	-0.049	0.30	-1.597	-1.808	-0.40	-0.40	P
24	1909 02 19.00796	07 42 44.348	13 22 18.03	-0.039	0.18	-1.539	-1.848	3.81	3.81	P
24	1909 02 18.97747	07 42 45.348	13 22 07.43	-0.039	0.18	1.103	0.794	0.34	0.34	P
24	1909 02 18.94614	07 42 46.158	13 22 01.63	-0.039	0.18	0.479	0.170	1.89	1.89	P
24	1907 11 05.87260	01 38 03.755	05 44 37.33	-0.009	-0.06	-0.070	-0.447	1.84	1.84	P
24	1907 11 05.80239	01 38 06.465	05 44 57.33	-0.009	-0.06	-1.756	-2.133	0.40	0.40	P
30	1906 07 30.03393	20 37 08.271	-10 22 41.86	-0.001	0.31	1.084	0.802	-0.20	-0.20	R
30	1906 07 30.03393	20 37 08.075	-10 22 42.04	-0.000	-0.00	-1.843	-2.125	-0.39	-0.39	V
136	1906 07 29.81450	20 37 18.263	-10 21 52.59	-0.001	0.31	0.414	0.132	0.80	0.80	R
136	1906 07 24.84288	20 41 01.158	-10 04 18.18	-0.001	0.31	0.329	0.048	1.15	1.15	R
136	1906 07 23.86711	20 41 44.095	-10 01 04.38	-0.001	0.31	-1.783	-2.064	0.13	0.13	R
136	1906 07 22.84903	20 42 28.969	-09 57 44.38	-0.001	0.31	1.546	1.266	1.49	1.49	R
30	1906 07 22.01271	20 43 05.107	-09 55 06.52	-0.001	0.31	-1.177	-1.457	-0.55	-0.55	R
30	1906 07 20.98651	20 43 49.657	-09 51 55.40	-0.000	-0.00	2.886	2.607	-0.49	-0.49	V
136	1906 07 20.85961	20 43 54.967	-09 51 31.27	-0.001	0.31	0.715	0.436	0.77	0.77	R
30	1906 07 20.04031	20 44 30.105	-09 49 03.64	-0.000	-0.00	4.923	4.644	-0.22	-0.22	V
136	1906 07 19.91950	20 44 34.955	-09 48 43.31	-0.000	-0.00	0.466	0.187	-1.02	-1.02	V
136	1906 07 19.89506	20 44 35.990	-09 48 37.82	-0.001	0.31	0.088	-0.191	0.11	0.11	R
30	1906 07 19.01757	20 45 13.159	-09 46 04.26	-0.000	-0.00	3.112	2.834	-0.95	-0.95	V
30	1906 07 19.01757	20 45 13.059	-09 46 03.60	-0.001	0.31	1.622	1.344	-0.29	-0.29	R
30	1906 07 18.00481	20 45 55.452	-09 43 11.50	-0.000	-0.00	4.585	4.307	-1.09	-1.09	V
30	1906 07 18.00481	20 45 55.223	-09 43 08.96	-0.001	0.31	1.142	0.865	1.45	1.45	R
30	1906 07 17.00789	20 46 36.119	-09 40 28.70	-0.001	0.31	0.420	0.143	-3.01	-3.01	R
24	1906 07 16.98299	20 46 36.814	-09 40 21.45	0.025	0.49	-4.862	-5.139	0.39	0.39	P
24	1906 07 16.92567	20 46 38.933	-09 40 11.45	0.025	0.49	-8.929	-9.205	1.02	1.02	P
24	1905 06 22.93796	15 28 50.564	-11 11 41.76	-0.022	-0.04	-1.560	-1.765	-0.30	-0.30	P
24	1905 06 22.93790	15 28 50.782	-11 11 38.86	-0.022	-0.04	1.711	1.506	2.60	2.60	P
30	1905 06 08.88361	15 36 11.423	-11 30 16.51	-0.022	-0.01	-0.746	-0.963	-2.77	-2.77	R
30	1905 06 02.89504	15 40 10.743	-11 43 47.29	-0.022	-0.01	-3.224	-3.444	-1.72	-1.72	R
30	1905 06 02.89504	15 40 10.826	-11 43 46.97	-0.022	-0.01	-1.972	-2.192	-1.40	-1.40	R
30	1905 06 01.90793	15 40 52.591	-11 46 19.28	-0.022	-0.01	-1.058	-1.278	-2.42	-2.42	R
30	1905 06 01.90793	15 40 52.663	-11 46 19.06	-0.022	-0.01	0.036	-0.185	-2.20	-2.20	R
30	1905 05 31.93091	15 41 34.409	-11 48 51.48	-0.022	-0.01	-0.662	-0.883	-0.30	-0.30	R

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30	1905 05 31.93091	15 41 34.413	-11 48 52.05	-0.022	-0.01	-0.612	-0.833	-0.88	-0.88	R
30	1905 05 30.93511	15 42 17.621	-11 51 33.19	-0.022	-0.01	0.731	0.510	-0.19	-0.19	R
30	1905 05 30.93511	15 42 17.584	-11 51 33.97	-0.022	-0.01	0.178	-0.043	-0.97	-0.97	R
30	1905 05 29.91933	15 43 02.095	-11 54 26.13	-0.022	-0.01	0.518	0.296	-3.45	-3.45	R
30	1905 05 29.91933	15 43 02.059	-11 54 26.11	-0.022	-0.01	-0.035	-0.256	-3.44	-3.44	R
136	1905 05 27.86448	15 44 33.412	-12 00 18.63	-0.011	0.04	2.143	1.921	1.26	1.26	R
84	1905 05 25.90134	15 46 01.329	-12 06 17.32	-0.011	0.04	-7.357	-7.580	-0.37	-0.37	R
84	1905 05 19.93950	15 50 35.438	-12 25 44.16	-0.011	0.04	1.457	1.233	1.04	1.04	R
136	1905 05 13.81899	15 55 16.362	-12 47 29.53	-0.011	0.04	3.098	2.875	-0.59	-0.59	R
136	1905 05 11.84157	15 56 45.443	-12 54 47.40	-0.011	0.04	3.237	3.014	-0.61	-0.61	R
136	1905 05 10.86674	15 57 28.702	-12 58 25.51	-0.011	0.04	0.871	0.649	-0.62	-0.62	R
24	1905 05 08.07407	15 59 30.817	-13 08 59.51	-0.010	0.01	0.325	0.104	-3.55	-3.55	P
24	1905 05 08.00913	15 59 33.678	-13 09 11.11	-0.010	0.01	0.232	0.011	-0.34	-0.34	P
24	1904 03 20.87885	10 41 51.866	05 00 05.43	-0.042	0.18	0.618	0.358	1.05	1.05	P
24	1904 03 20.81982	10 41 53.856	04 59 44.03	-0.042	0.18	-2.163	-2.424	0.61	0.61	P
84	1904 03 05.95080	10 51 54.096	03 26 51.67	-0.040	0.04	-1.647	-1.914	0.70	0.70	R
84	1904 03 05.95080	10 51 54.096	03 26 51.23	-0.042	0.18	-1.647	-1.914	0.25	0.25	R
84	1904 03 05.93557	10 51 54.805	03 26 45.66	-0.040	0.04	-1.082	-1.350	0.51	0.51	R
84	1904 03 05.93557	10 51 54.815	03 26 45.21	-0.042	0.18	-0.930	-1.197	0.06	0.06	R
84	1904 02 25.94844	10 58 23.543	02 30 37.04	-0.042	0.18	-0.840	-1.106	0.52	0.52	R
7	1902 12 02.95635	04 54 21.685	13 28 19.57	-0.000	-0.00	2.980	2.595	0.29	0.29	V
7	1902 12 02.95354	04 54 21.875	13 28 20.55	-0.000	-0.00	3.641	3.256	0.88	0.88	V
-0	1902 12 01.79759	04 55 20.009	13 31 05.26	-0.000	-0.00	-2.290	-2.675	-3.32	-3.32	R
-0	1902 11 30.92141	04 56 04.100	13 33 23.53	-0.000	-0.00	-0.401	-0.786	2.74	2.74	R
-0	1902 11 30.90358	04 56 04.785	13 33 20.74	-0.000	-0.00	-3.830	-4.215	-2.74	-2.74	R
15	1902 11 28.93222	04 57 43.202	13 38 25.13	-0.000	-0.00	-0.722	-1.106	-5.43	-5.43	R
84	1902 11 26.97956	04 59 18.895	13 43 48.47	-0.000	-0.00	1.466	1.084	1.31	1.31	P
30	1902 11 24.86859	05 01 00.158	13 49 44.39	-0.051	0.17	0.638	0.257	0.59	0.59	R
30	1902 11 24.86859	05 01 00.090	13 49 44.19	-0.051	0.17	-0.376	-0.757	0.38	0.38	R
30	1902 11 23.87439	05 01 46.879	13 52 36.02	-0.051	0.17	1.813	1.433	0.21	0.21	R
30	1902 11 23.87439	05 01 46.816	13 52 35.54	-0.051	0.17	0.868	0.488	-0.28	-0.28	R
30	1902 11 22.87439	05 02 33.067	13 55 30.60	-0.051	0.17	1.382	1.003	-0.83	-0.83	R
794	1902 11 21.11321	05 03 52.401	14 00 46.39	-0.051	0.17	-1.506	-1.883	-0.45	-0.45	R
794	1902 11 20.09963	05 04 37.054	14 03 52.41	-0.051	0.17	-0.872	-1.249	1.00	1.00	R
999	1901 09 29.92066	22 50 07.277	-04 44 37.90	0.035	0.36	1.248	0.932	0.54	0.54	R
999	1901 09 28.87592	22 50 41.090	-04 38 49.48	0.035	0.36	0.181	-0.136	-0.13	-0.13	R
8	1901 09 25.85183	22 52 24.775	-04 21 28.66	0.035	0.36	10.159	9.838	1.17	1.17	R
8	1901 09 24.87607	22 52 59.240	-04 15 44.95	0.035	0.36	6.719	6.398	1.53	1.53	R
8	1901 09 23.87489	22 53 35.184	-04 09 49.42	0.035	0.36	0.889	0.567	1.04	1.04	R
30	1901 09 20.88920	22 55 27.587	-03 51 49.83	0.038	0.31	4.548	4.224	-0.31	-0.31	R
30	1901 09 17.86609	22 57 26.148	-03 33 10.96	0.038	0.31	2.245	1.919	0.17	0.17	R
30	1901 09 17.86609	22 57 26.177	-03 33 10.72	0.038	0.31	2.676	2.350	0.41	0.41	R
8	1901 09 14.90003	22 59 26.184	-03 14 39.13	0.038	0.31	-1.521	-1.849	-0.24	-0.24	R
8	1901 09 09.95647	23 02 51.792	-02 43 38.70	0.038	0.31	-2.141	-2.470	-1.13	-1.13	R
30	1901 09 09.90752	23 02 54.129	-02 43 17.46	0.038	0.31	1.669	1.341	2.10	2.10	R
30	1901 09 08.86233	23 03 37.878	-02 36 47.83	0.038	0.31	-1.277	-1.605	0.17	0.17	R
999	1901 09 07.94059	23 04 16.432	-02 31 04.63	0.038	0.31	-2.942	-3.270	-0.21	-0.21	R
45	1890 08 10.91940	23 21 40.667	-00 02 05.72	-0.015	0.27	-1.336	-1.600	3.60	3.60	R
45	1890 08 09.93442	23 22 06.223	00 01 26.87	-0.015	0.27	-0.652	-0.915	1.84	1.84	R
-1	1888 04 15.92417	13 18 51.881	-05 25 23.26	-0.010	0.31	-1.680	-1.900	-1.74	-1.74	R

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534	1871 03 07.87783	10 51 14.036	03 28 39.07	-0.000	-0.00	-6.275	-6.420	-1.66	-1.66	R
-1	1871 03 06.85245	10 51 58.117	03 22 11.14	-0.039	0.06	-5.121	-5.266	-0.08	-0.08	R
534	1871 03 04.87870	10 53 23.620	03 09 41.37	-0.000	-0.00	-1.107	-1.252	-1.33	-1.33	R
534	1871 03 03.98965	10 54 02.006	03 04 03.90	-0.000	-0.00	-2.246	-2.391	-2.64	-2.64	R
0	1871 03 03.00653	10 54 44.774	02 58 01.22	-0.000	-0.00	-1.994	-2.139	5.31	5.31	M
534	1871 03 02.98660	10 54 45.855	02 57 55.44	-0.000	-0.00	1.453	1.308	7.02	7.02	R
534	1871 03 02.90192	10 54 49.550	02 57 17.52	-0.041	0.20	0.267	0.122	0.98	0.98	R
534	1871 03 01.97709	10 55 29.794	02 51 19.50	-0.041	0.20	1.653	1.508	-9.96	-9.96	R
-1	1871 03 01.95685	10 55 30.467	02 51 21.70	-0.041	0.20	-1.733	-1.878	-0.13	-0.13	R
39	1871 03 01.82730	10 55 36.164	02 50 31.99	-0.041	0.20	-2.422	-2.567	-1.14	-1.14	R
-1	1871 02 28.98980	10 56 12.559	02 45 22.22	-0.041	0.20	-0.637	-0.782	1.53	1.53	R
39	1871 02 28.96341	10 56 13.593	02 45 09.96	-0.041	0.20	-2.743	-2.888	-0.75	-0.75	R
39	1871 02 25.98563	10 58 22.917	02 26 54.21	-0.041	0.20	3.478	3.334	0.59	0.59	R
0	1869 12 15.95995	04 45 20.684	13 07 05.01	-0.000	-0.00	2.555	2.311	4.42	4.42	M
558	1869 12 10.92893	04 49 26.233	13 14 29.37	-0.038	0.14	2.658	2.413	-1.36	-1.36	R
558	1869 12 09.91330	04 50 17.108	13 16 14.53	-0.038	0.14	1.669	1.423	-3.05	-3.05	R
534	1869 12 08.87086	04 51 09.628	13 18 13.10	-0.038	0.14	-0.558	-0.804	0.53	0.53	R
534	1869 12 07.98344	04 51 54.716	13 19 54.26	-0.038	0.14	3.135	2.889	-0.48	-0.48	R
534	1869 12 07.97894	04 51 55.176	13 19 55.11	-0.000	-0.00	6.512	6.266	-0.16	-0.16	V
7	1868 09 12.96259	22 48 33.631	-03 49 54.34	-0.000	-0.00	-3.557	-3.948	5.56	5.56	M
-3	1868 09 10.97694	22 49 55.440	-03 37 36.71	0.021	0.15	5.630	5.238	-0.37	-0.37	R
39	1868 09 10.85782	22 50 00.416	-03 36 50.17	0.021	0.15	5.465	5.073	1.51	1.51	R
7	1868 09 09.97219	22 50 36.272	-03 31 12.89	-0.000	-0.00	-4.930	-5.322	6.30	6.30	M
39	1868 09 09.92796	22 50 38.878	-03 31 02.15	0.021	0.15	6.425	6.032	0.72	0.72	R
39	1868 09 08.93837	22 51 19.438	-03 24 52.83	0.021	0.15	-2.457	-2.850	-1.09	-1.09	R
558	1868 09 08.91479	22 51 20.584	-03 24 44.26	0.021	0.15	-0.040	-0.433	-1.50	-1.50	R
7	1868 09 07.97863	22 51 59.540	-03 18 46.13	-0.000	-0.00	-2.298	-2.691	5.78	5.78	M
39	1868 09 07.93282	22 52 01.627	-03 18 36.41	0.021	0.15	-0.012	-0.405	-1.37	-1.37	R
558	1868 09 07.91565	22 52 02.404	-03 18 29.14	0.021	0.15	0.924	0.531	-0.67	-0.67	R
39	1868 09 06.88698	22 52 45.293	-03 12 05.34	0.021	0.15	-3.595	-3.988	-1.28	-1.28	R
39	1868 09 05.90296	22 53 26.857	-03 05 57.46	0.021	0.15	-0.516	-0.909	-0.17	-0.17	R
7	1868 09 04.98827	22 54 05.223	-03 00 12.53	-0.000	-0.00	-1.844	-2.237	4.78	4.78	M
7	1868 09 03.99149	22 54 47.458	-02 54 04.93	-0.000	-0.00	2.026	1.633	3.82	3.82	M
39	1868 08 29.95504	22 58 18.124	-02 23 41.86	-0.000	-0.00	-0.479	-0.871	2.36	2.36	V
13	1868 08 26.01441	23 00 59.156	-02 00 58.96	-0.000	-0.00	1.588	1.199	-0.95	-0.95	M
534	1867 07 04.96458	17 50 05.586	-14 14 19.66	0.011	0.06	1.590	1.278	0.36	0.36	R
13	1867 06 29.95906	17 53 50.901	-14 11 58.73	-0.000	-0.00	-1.164	-1.479	-1.33	-1.33	M
13	1867 06 28.96232	17 54 37.248	-14 11 41.79	-0.000	-0.00	0.753	0.438	-2.84	-2.84	M
13	1867 06 27.96560	17 55 23.731	-14 11 24.14	-0.000	-0.00	-0.325	-0.640	-0.29	-0.29	M
-1	1867 06 26.98205	17 56 09.903	-14 11 13.25	0.011	0.06	-0.556	-0.872	-1.04	-1.04	R
13	1867 06 25.97214	17 56 57.559	-14 11 04.23	-0.000	-0.00	-2.020	-2.335	-0.58	-0.58	M
-1	1867 06 24.01633	17 58 30.597	-14 10 58.87	0.011	0.06	0.083	-0.233	-2.03	-2.03	R
534	1867 06 21.96558	18 00 08.641	-14 11 01.35	0.011	0.06	0.590	0.274	2.24	2.24	R
534	1867 06 21.95696	18 00 08.997	-14 11 02.80	0.011	0.06	-0.382	-0.698	0.84	0.84	R
534	1867 06 20.96680	18 00 56.466	-14 11 11.37	0.011	0.06	1.347	1.031	0.64	0.64	R
534	1867 06 19.96819	18 01 44.055	-14 11 24.54	0.011	0.06	-1.084	-1.400	-0.75	-0.75	R
-6	1867 06 14.23716	18 06 16.037	-14 13 32.64	-0.000	-0.00	4.333	4.018	-0.02	-0.02	M
-6	1867 06 12.24369	18 07 48.404	-14 14 45.38	-0.000	-0.00	1.287	0.973	-1.17	-1.17	M
-6	1867 06 11.24703	18 08 34.049	-14 15 26.83	-0.000	-0.00	-0.029	-0.343	-1.92	-1.92	M
-6	1867 06 07.25994	18 11 32.487	-14 18 41.23	-0.000	-0.00	1.515	1.203	-1.35	-1.35	M

OBS	DATE	R.A.	DEC.	FK4-CAT. R.A.	FK4-CAT. DEC.	(O-C) R.A. BEFORE	(O-C) R.A. AFTER	(O-C) DEC. BEFORE	(O-C) DEC. AFTER	TYPE
		H M S	O / //	S	//	//	//	//	//	
7	1866 05 02.92487	13 07 47.736	-04 01 44.89	-0.000	-0.00	0.345	0.161	1.32	1.32	M
0	1866 04 25.95328	13 11 52.527	-04 35 03.21	-0.000	-0.00	1.778	1.590	4.32	4.32	M
13	1866 04 25.94086	13 11 52.828	-04 35 09.80	-0.000	-0.00	-0.700	-0.888	1.57	1.57	M
0	1866 04 24.95644	13 12 30.295	-04 40 14.60	-0.000	-0.00	1.710	1.521	1.97	1.97	M
0	1866 04 21.96598	13 14 27.109	-04 56 06.43	-0.000	-0.00	1.895	1.706	6.89	6.89	M
0	1866 04 20.96917	13 15 07.173	-05 01 36.37	-0.000	-0.00	3.187	2.997	4.93	4.93	M
0	1866 04 18.97557	13 16 28.494	-05 12 47.24	-0.000	-0.00	3.515	3.324	2.07	2.07	M
0	1866 04 17.97877	13 17 09.567	-05 18 20.50	-0.000	-0.00	0.996	0.805	8.31	8.31	M
13	1866 04 15.97278	13 18 33.669	-05 30 01.02	-0.000	-0.00	2.850	2.659	0.65	0.65	M
13	1866 04 14.97599	13 19 15.462	-05 35 50.15	-0.000	-0.00	-2.187	-2.378	-0.02	-0.02	M
58	1866 04 14.87678	13 19 19.969	-05 36 24.15	-0.025	0.17	1.557	1.365	0.87	0.87	R
-5	1866 04 12.84920	13 20 46.222	-05 48 23.45	-0.025	0.17	-0.075	-0.266	-2.52	-2.52	R
534	1866 04 11.98385	13 21 23.144	-05 53 31.14	-0.025	0.17	0.712	0.520	-2.07	-2.07	R
13	1866 04 09.98517	13 22 48.414	-06 05 21.72	-0.000	-0.00	-4.487	-4.678	2.42	2.42	M
534	1866 04 09.93376	13 22 50.948	-06 05 44.24	-0.025	0.17	0.055	-0.136	-1.69	-1.69	R
58	1866 04 09.89619	13 22 52.587	-06 05 55.59	-0.025	0.17	0.334	0.143	0.55	0.55	R
-1	1866 04 09.89104	13 22 52.712	-06 05 59.06	-0.025	0.17	-1.477	-1.668	-1.17	-1.17	R
-5	1866 04 08.93176	13 23 33.926	-06 11 48.58	-0.025	0.17	1.383	1.192	-6.29	-6.29	R
58	1866 04 08.87588	13 23 36.196	-06 12 05.56	-0.025	0.17	-0.976	-1.168	-2.96	-2.96	R
58	1866 04 07.83751	13 24 20.621	-06 18 15.21	-0.025	0.17	-0.630	-0.822	0.81	0.81	R
13	1866 04 06.00498	13 25 38.593	-06 29 13.76	-0.000	-0.00	1.051	0.860	1.42	1.42	M
-1	1866 04 05.98333	13 25 39.449	-06 29 24.70	-0.025	0.17	0.190	-0.001	-1.71	-1.71	R
-1	1866 04 03.97092	13 27 04.243	-06 41 26.76	-0.025	0.17	-0.604	-0.795	-1.61	-1.61	R
0	1865 02 20.87120	07 39 20.497	13 33 58.22	-0.000	-0.00	9.708	9.500	-8.40	-8.40	M
-1	1865 02 06.84299	07 46 22.809	12 38 32.58	-0.050	0.19	0.254	0.033	-0.12	-0.12	R
-1	1865 02 05.84569	07 47 01.543	12 34 32.09	-0.050	0.19	-1.966	-2.188	1.24	1.24	R
-1	1865 02 04.89872	07 47 39.400	12 30 39.53	-0.050	0.19	-0.963	-1.186	-1.98	-1.98	R
0	1865 01 28.96884	07 52 40.682	12 03 07.73	-0.000	-0.00	2.442	2.214	1.46	1.46	M
13	1865 01 28.95643	07 52 41.219	12 03 02.25	-0.000	-0.00	1.910	1.683	-1.07	-1.07	M
-6	1865 01 27.11798	07 54 07.350	11 55 54.32	-0.050	0.19	5.844	5.616	-3.10	-3.10	R
-6	1865 01 26.08437	07 54 56.048	11 52 01.09	-0.050	0.19	-0.030	-0.258	0.30	0.30	R
-6	1865 01 21.12219	07 58 57.346	11 33 45.38	-0.050	0.19	-2.312	-2.541	-2.44	-2.44	R
13	1865 01 05.03533	08 11 57.333	10 47 04.11	-0.000	-0.00	-0.712	-0.936	1.69	1.69	M
0	1863 11 20.88505	01 16 22.728	03 53 22.80	-0.000	-0.00	-0.516	-0.925	3.36	3.36	M
534	1863 11 03.88618	01 24 26.737	05 00 37.53	-0.007	0.07	-2.315	-2.756	-0.25	-0.25	R
-4	1863 10 20.80710	01 34 04.480	06 21 34.19	-0.007	0.07	-0.717	-1.170	-0.16	-0.16	R
-4	1863 10 19.82385	01 34 47.491	06 27 42.71	-0.007	0.07	-0.242	-0.696	0.34	0.34	R
-4	1863 10 19.77798	01 34 49.597	06 28 01.61	-0.007	0.07	0.665	0.211	2.09	2.09	R
-4	1863 10 16.85514	01 36 57.738	06 46 22.20	-0.007	0.07	0.939	0.485	-2.22	-2.22	R
-6	1863 10 15.95264	01 37 37.144	06 52 04.70	-0.000	-0.00	0.572	0.118	-3.09	-3.09	M
534	1863 10 15.91006	01 37 39.065	06 52 23.58	-0.000	-0.00	0.471	0.017	-0.23	-0.23	R
0	1863 10 15.00122	01 38 18.926	06 58 12.82	-0.000	-0.00	3.394	2.941	2.69	2.69	M
-5	1863 10 14.95588	01 38 20.812	06 58 28.82	-0.000	-0.00	1.983	1.529	1.22	1.22	M
-6	1863 10 14.13604	01 38 56.460	07 03 40.03	-0.000	-0.00	-0.874	-1.328	-0.74	-0.74	R
-5	1863 10 13.95911	01 39 04.156	07 04 45.88	-0.000	-0.00	0.927	0.473	-1.84	-1.84	M
-6	1863 10 13.09850	01 39 41.513	07 10 14.35	-0.000	-0.00	-0.916	-1.369	-1.98	-1.98	R
534	1863 10 11.98478	01 40 29.359	07 17 20.39	-0.007	0.07	-0.638	-1.091	0.46	0.46	R
-5	1863 10 10.96878	01 41 13.158	07 23 44.69	-0.000	-0.00	4.504	4.052	-1.57	-1.57	M
-6	1863 10 10.12481	01 41 48.619	07 29 04.69	-0.007	0.07	-2.790	-3.242	-1.94	-1.94	R
-5	1863 10 09.97199	01 41 55.092	07 30 04.61	-0.000	-0.00	-0.282	-0.734	0.65	0.65	M

OBS	DATE	R.A. H M S	DEC. ° ′ ″	FK4-CAT. R.A. S	FK4-CAT. DEC. ″	(O-C) R.A. BEFORE ″	(O-C) R.A. AFTER ″	(O-C) DEC. BEFORE ″	(O-C) DEC. AFTER ″	TYPE
-6	1863 10 09.91261	01 41 57.780	07 30 21.50	-0.000	-0.00	1.368	0.916	-4.89	-4.89	R
-6	1863 10 09.14918	01 42 29.762	07 35 14.14	-0.007	0.07	-0.518	-0.969	-1.00	-1.00	R
534	1863 10 08.97434	01 42 36.714	07 36 21.65	-0.007	0.07	-4.241	-4.692	1.35	1.35	R
534	1863 10 07.92048	01 43 20.583	07 42 54.84	-0.007	0.07	-2.991	-3.442	-1.14	-1.14	R
534	1863 10 07.86121	01 43 23.131	07 43 16.51	-0.007	0.07	-2.261	-2.712	-1.58	-1.58	R
534	1863 10 04.04663	01 45 55.682	08 06 47.57	-0.007	0.07	-4.411	-4.859	1.04	1.04	R
0	1863 09 30.04916	01 48 25.627	08 30 31.25	-0.000	-0.00	3.530	3.087	2.83	2.83	M
0	1862 08 25.90140	19 58 21.556	-13 01 57.52	-0.000	-0.00	2.154	1.750	-2.96	-2.96	M
0	1862 08 19.91984	20 01 18.477	-12 39 13.99	-0.000	-0.00	2.389	1.976	3.26	3.26	M
0	1862 08 07.95783	20 08 49.822	-11 52 59.11	-0.000	-0.00	2.702	2.277	-0.52	-0.52	M
0	1862 07 31.98048	20 13 54.407	-11 26 43.48	-0.000	-0.00	1.551	1.122	2.20	2.20	M
793	1862 07 31.18800	20 14 30.159	-11 23 50.88	-0.000	-0.00	1.622	1.192	1.50	1.50	M
-5	1862 07 28.94487	20 16 12.295	-11 15 53.86	-0.000	-0.00	4.407	3.977	-2.54	-2.54	M
793	1862 07 26.20428	20 18 17.356	-11 06 23.31	-0.000	-0.00	-1.914	-2.345	-0.98	-0.98	M
-1	1862 07 25.98251	20 18 27.561	-11 05 39.93	0.031	-0.12	-1.099	-1.529	-2.25	-2.25	R
-5	1862 07 25.95468	20 18 29.004	-11 05 36.12	-0.000	-0.00	1.109	0.678	-4.21	-4.21	M
-1	1862 07 25.01731	20 19 11.637	-11 02 24.10	0.031	-0.12	-3.461	-3.891	-0.33	-0.33	R
-1	1862 07 23.01139	20 20 43.670	-10 55 53.36	0.031	-0.12	-0.452	-0.881	-1.92	-1.92	R
0	1862 07 20.01955	20 22 59.662	-10 46 31.09	-0.000	-0.00	1.188	0.760	4.26	4.26	M
-1	1861 06 13.97273	15 18 38.246	-10 54 37.98	-0.018	0.36	6.015	5.740	-0.32	-0.32	R
0	1860 04 17.85879	10 26 01.098	07 22 14.23	-0.000	-0.00	2.885	2.682	-3.23	-3.23	M
0	1860 04 07.88696	10 27 15.370	06 51 28.82	-0.000	-0.00	5.146	4.932	-1.90	-1.90	M
0	1860 03 23.93181	10 32 51.038	05 42 26.38	-0.000	-0.00	1.755	1.525	2.19	2.19	M
58	1860 03 21.81176	10 33 58.468	05 30 40.75	-0.038	0.14	2.067	1.835	-1.14	-1.14	R
-5	1860 03 16.83730	10 36 52.418	05 01 48.44	-0.039	0.13	2.532	2.296	-2.03	-2.03	R
-5	1860 03 13.88291	10 38 44.946	04 43 57.59	-0.039	0.13	4.495	4.258	0.64	0.64	R
-5	1860 03 13.88291	10 38 44.916	04 43 58.80	-0.039	0.13	4.042	3.804	1.86	1.86	R
-5	1860 03 13.86077	10 38 45.353	04 43 47.88	-0.039	0.13	-2.742	-2.980	-0.91	-0.91	R
-5	1860 03 12.84014	10 39 25.817	04 37 29.52	-0.039	0.13	0.755	0.517	-2.34	-2.34	R
-5	1860 03 12.81611	10 39 26.804	04 37 20.58	-0.039	0.13	0.904	0.666	-2.36	-2.36	R
-5	1860 03 11.97217	10 40 00.117	04 32 02.97	-0.039	0.13	-2.789	-3.027	-6.38	-6.38	R
-5	1860 03 11.97217	10 40 00.186	04 32 04.48	-0.039	0.13	-1.746	-1.984	-4.87	-4.87	R
-5	1860 03 11.94527	10 40 01.654	04 31 57.90	-0.039	0.13	3.600	3.362	-1.44	-1.44	R
-5	1860 03 11.94527	10 40 01.564	04 31 56.22	-0.039	0.13	2.237	1.998	-3.12	-3.12	R
58	1860 03 09.85322	10 41 26.656	04 18 53.63	-0.039	0.13	-2.204	-2.443	-1.63	-1.63	R
0	1860 03 02.99849	10 46 18.699	03 35 42.87	-0.000	-0.00	3.230	2.990	4.66	4.66	M
0	1860 03 01.00496	10 47 45.618	03 23 08.93	-0.000	-0.00	2.921	2.680	3.88	3.88	M
-5	1860 02 29.87678	10 47 50.994	03 22 16.62	-0.039	0.11	-1.887	-2.128	-0.32	-0.32	R
802	1860 02 27.21373	10 49 47.312	03 05 43.89	-0.042	0.19	-2.853	-3.093	2.22	2.22	R
-5	1860 02 26.96945	10 49 58.275	03 04 11.02	-0.000	-0.00	1.445	1.205	0.21	0.21	M
-6	1860 02 25.97263	10 50 42.005	02 58 08.71	-0.000	-0.00	5.043	4.804	5.70	5.70	M
58	1860 02 25.95802	10 50 42.183	02 57 58.17	-0.042	0.19	-1.881	-2.121	0.85	0.85	R
58	1860 02 25.93316	10 50 43.489	02 57 48.52	-0.000	-0.00	1.082	0.842	-0.00	-0.00	V
0	1860 02 14.05641	10 58 57.353	01 50 13.12	-0.000	-0.00	2.612	2.378	3.17	3.17	M
-1	1859 01 08.97546	04 15 13.036	12 41 04.57	-0.001	0.20	-4.206	-4.610	-2.82	-2.82	R
-1	1859 01 05.93806	04 16 06.675	12 38 12.50	-0.000	-0.00	0.107	-0.304	1.59	1.59	V
-1	1858 12 18.89619	04 25 39.671	12 40 38.00	0.000	0.34	-2.495	-2.943	-1.03	-1.03	R
-1	1858 12 17.98716	04 26 18.230	12 41 40.94	-0.000	-0.00	-2.999	-3.449	0.18	0.18	R
-1	1858 09 14.98680	04 53 58.677	17 08 12.92	0.009	0.22	-0.616	-0.952	0.85	0.85	R
-1	1858 02 10.75992	23 58 38.615	-00 33 51.35	0.001	0.34	-0.083	-0.369	-0.30	-0.30	R

OBS	DATE	R.A.	DEC.	FK4-CAT. R.A.	FK4-CAT. DEC.	(O-C) R.A. BEFORE	(O-C) R.A. AFTER	(O-C) DEC. BEFORE	(O-C) DEC. AFTER	TYPE
		H M S	O / //	S	//	//	//	//	//	
-1	1858 02 07.74584	23 54 35.128	-00 56 57.83	0.001	0.34	-0.141	-0.430	1.32	1.32	R
-1	1858 02 02.76487	23 47 58.052	-01 34 30.58	0.019	0.05	0.437	0.146	0.41	0.41	R
-1	1857 12 12.83180	22 48 27.351	-06 43 52.20	0.033	0.32	1.051	0.708	-2.46	-2.46	R
-1	1857 11 19.85525	22 31 04.725	-07 47 52.37	0.033	0.32	-1.698	-2.079	0.36	0.36	R
-1	1857 11 15.81325	22 28 52.159	-07 52 58.61	0.033	0.32	-1.192	-1.581	-2.09	-2.09	R
-1	1857 11 02.86539	22 23 48.647	-07 55 10.80	0.033	0.32	-2.481	-2.898	1.32	1.32	R
-1	1857 10 24.92070	22 22 15.408	-07 43 53.09	0.033	0.32	-5.326	-5.763	-2.09	-2.09	R
-5	1857 10 21.81061	22 22 07.287	-07 37 18.11	0.033	0.32	1.365	0.920	3.43	3.43	R
-5	1857 10 20.85332	22 22 07.312	-07 35 02.82	0.033	0.32	4.741	4.294	2.98	2.98	R
-1	1857 10 18.87153	22 22 10.633	-07 30 01.32	0.033	0.32	3.113	2.661	0.03	0.03	R
520	1857 10 17.85326	22 22 14.595	-07 27 14.41	0.033	0.32	6.033	5.579	-1.91	-1.91	R
520	1857 10 15.84667	22 22 25.732	-07 21 17.11	0.033	0.32	3.968	3.508	-1.46	-1.46	R
520	1857 10 14.84986	22 22 33.436	-07 18 09.59	0.033	0.32	6.851	6.389	-3.02	-3.02	R
-1	1857 10 13.81078	22 22 42.524	-07 14 41.71	0.033	0.32	5.156	4.692	-0.42	-0.42	R
-1	1857 10 06.89990	22 24 17.368	-06 48 29.55	0.033	0.32	-0.224	-0.704	-0.21	-0.21	R
-1	1857 10 04.98159	22 24 54.053	-06 40 18.51	0.033	0.32	1.020	0.536	-6.06	-6.07	R
-1	1857 09 29.93009	22 26 50.430	-06 16 26.96	0.033	0.32	-3.253	-3.748	1.86	1.86	R
-1	1857 09 29.01391	22 27 14.488	-06 11 55.68	0.033	0.32	-3.446	-3.943	-2.02	-2.02	R
520	1857 09 28.01676	22 27 41.881	-06 06 45.37	0.033	0.30	-1.671	-2.170	3.34	3.34	R

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